8.8 Filters Using Coupled Resonators

Another type of bandpass filter that can be conveniently fabricated in microstrip or stripline form is the capacitive-gap coupled resonator filter shown in Figure 8.50. An \( N \)th-order filter of this form will use \( N \) resonant series sections of transmission line with \( N + 1 \) capacitive gaps between them. These gaps can be approximated as series capacitors; design data relating the capacitance to the gap size and transmission line parameters is given in graphical form in reference [1]. The filter can then be modeled as shown in Figure 8.50b. The resonators are approximately \( \lambda/2 \) long at the center frequency, \( \omega_0 \).

Next, we redraw the equivalent circuit of Figure 8.50b with negative-length transmission line sections on either side of the series capacitors. The lines of length \( \phi \) will be \( \lambda/2 \) long at \( \omega_0 \), so the electrical length \( \theta_i \) of the \( i \)th section in Figures 8.50a, b is

\[
\theta_i = \pi + \frac{1}{2} \phi_i + \frac{1}{2} \phi_{i+1} \quad \text{for } i = 1, 2, \ldots, N,
\]

with \( \phi_i < 0 \). The reason for doing this is that the combination of series capacitor and negative-length transmission lines forms the equivalent circuit of an admittance inverter, as seen from Figure 8.38c. In order for this equivalence to be valid, the following relationship
must hold between the electrical length of the lines and the capacitive susceptance:

\[ \phi_i = -\tan^{-1}(2Z_0B_i). \]  

(8.133)

Then the resulting inverter constant can be related to the capacitive susceptance as

\[ B_i = \frac{J_i}{1 - (Z_0J_i)^2}. \]  

(8.134)

(These results are given in Figure 8.38, and their derivation is requested in Problem 8.14.)

The capacitive-gap coupled filter can then be modeled as shown in Figure 8.50d. Now consider the equivalent circuit shown in Figure 8.45b for a coupled line bandpass filter. Since these two circuits are identical (as \( \phi = 2\theta = \pi \) at the center frequency), we can use the results from the coupled line filter analysis to complete the present problem. Thus, we can use (8.121) to find the admittance inverter constants, \( J_i \), from the low-pass prototype values, \( g_i \), and the fractional bandwidth, \( \Delta \). As in the case of the coupled line filter, there will be \( N + 1 \) inverter constants for an \( N \)th-order filter. Then (8.134) can be used to find the susceptance, \( B_i \), for the \( i \)th coupling gap. Finally, the electrical length of the resonator sections can be found from (8.132) and (8.133):

\[ \theta_i = \pi - \frac{1}{2} \left[ \tan^{-1}(2Z_0B_i) + \tan^{-1}(2Z_0B_{i+1}) \right]. \]  

(8.135)

**EXAMPLE 8.9  CAPACITIVELY COUPLED SERIES RESONATOR BANDPASS FILTER DESIGN**

Design a bandpass filter using capacitive coupled series resonators, with a 0.5 dB equal-ripple passband characteristic. The center frequency is 2.0 GHz, the bandwidth is 10\%, and the impedance is 50 \( \Omega \). At least 20 dB of attenuation is required at 2.2 GHz.

**Solution**

We first determine the order of the filter to satisfy the attenuation specification at 2.2 GHz. Using (8.71) to convert to normalized frequency gives

\[ \omega \leftarrow \frac{1}{\Delta} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{0.1} \left( \frac{2.2}{2.0} - \frac{2.0}{2.2} \right) = 1.91. \]

Then,

\[ \left| \frac{\omega_i}{\omega_c} \right| - 1 = 1.91 - 1.0 = 0.91. \]

From Figure 8.27a, we see that \( N = 3 \) should satisfy the attenuation specification at 2.2 GHz. The low-pass prototype values are given in Table 8.4, from which the inverter constants can be calculated using (8.121). Then the coupling susceptances can be found from (8.134), and the coupling capacitor values as

\[ C_n = \frac{B_n}{\omega_0}. \]

Finally, the resonator lengths can be calculated from (8.135). The following table summarizes these results.
8.8 Filters Using Coupled Resonators

### FIGURE 8.51
Amplitude response for the capacitive-gap coupled series resonator bandpass filter of Example 8.9.

<table>
<thead>
<tr>
<th>n</th>
<th>$g_n$</th>
<th>$Z_0 J_n$</th>
<th>$B_n$</th>
<th>$C_n$ (pF)</th>
<th>$\theta_n$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5963</td>
<td>0.3137</td>
<td>$6.96 \times 10^{-3}$</td>
<td>0.554</td>
<td>155.8</td>
</tr>
<tr>
<td>2</td>
<td>1.0967</td>
<td>0.1187</td>
<td>$2.41 \times 10^{-3}$</td>
<td>0.192</td>
<td>166.5</td>
</tr>
<tr>
<td>3</td>
<td>1.5963</td>
<td>0.1187</td>
<td>$2.41 \times 10^{-3}$</td>
<td>0.192</td>
<td>155.8</td>
</tr>
<tr>
<td>4</td>
<td>1.0000</td>
<td>0.3137</td>
<td>$6.96 \times 10^{-3}$</td>
<td>0.554</td>
<td>—</td>
</tr>
</tbody>
</table>

The calculated amplitude response is plotted in Figure 8.51. The specifications of this filter are the same as the coupled line bandpass filter of Example 8.8, and comparison of the results in Figures 8.51 and 8.46 shows that the responses are identical near the passband region.

### Bandpass Filters Using Capacitively Coupled Shunt Resonators

A related type of bandpass filter is shown in Figure 8.52, where short-circuited shunt resonators are capacitively coupled with series capacitors. An $N$th-order filter will use $N$ stubs, which are slightly shorter than $\lambda/4$ at the filter center frequency. The short-circuited stub resonators can be made from sections of coaxial line using ceramic materials having a very high dielectric constant and low loss, resulting in a very compact design even at UHF frequencies [9]. Such filters are often referred to as ceramic resonator filters and are

#### FIGURE 8.52
A bandpass filter using capacitively coupled shunt stub resonators.
among the most common types of RF bandpass filters used in portable wireless systems. Most cellular telephones, GPS receivers, and other wireless devices employ two or more filters of this type.

Operation and design of this filter can be understood by beginning with the general bandpass filter circuit of Figure 8.53a, where shunt \( LC \) resonators alternate with admittance inverters. As in the case of previous coupled resonator bandpass and bandstop filters, the function of the admittance inverters is to convert alternate shunt resonators to series resonators; the extra inverters at the ends serve to scale the impedance level of the filter to a realistic level. Using an analysis similar to that used for the bandstop filter, we can derive the admittance inverter constants as

\[
Z_{0J_01} = \sqrt{\frac{\pi \Delta}{4g_1}}, \tag{8.136a}
\]

\[
Z_{0J_n, n+1} = \frac{\pi \Delta}{4\sqrt{g_ng_{n+1}}}, \tag{8.136b}
\]

\[
Z_{0J_N, N+1} = \sqrt{\frac{\pi \Delta}{4g_Ng_{N+1}}}. \tag{8.136c}
\]

Similarly, the coupling capacitor values can be found as

\[
C_{01} = \frac{J_{01}}{\omega_0\sqrt{1 - (Z_{0J_01})^2}}, \tag{8.137a}
\]

\[
C_{n, n+1} = \frac{J_{n, n+1}}{\omega_0}, \tag{8.137b}
\]

\[
C_{N, N+1} = \frac{J_{N, N+1}}{\omega_0\sqrt{1 - (Z_{0J_N, N+1})^2}}. \tag{8.137c}
\]

Note that the end capacitors are treated differently than the internal elements.

Next, we replace the admittance inverters of Figure 8.53a with the equivalent \( \pi \)-network of Figure 8.38d, to produce the equivalent lumped-element circuit shown in Figure 8.53b. Note that the shunt capacitors of the admittance inverter circuits are negative, but these elements combine in parallel with the larger capacitor of the \( LC \) resonator to yield a net capacitance value that is positive. The resulting circuit is shown in Figure 8.53c, where the effective resonator capacitor values are given by

\[
C'_n = C_n + \Delta C_n = C_n - C_{n-1, n} - C_{n, n+1}, \tag{8.138}
\]

where \( \Delta C_n = -C_{n-1, n} - C_{n, n+1} \) represents the change in the resonator capacitance caused by the parallel addition of the inverter elements.

Finally, the shunt \( LC \) resonators of Figure 8.53c are replaced with short-circuited transmission stubs, as in the circuit of Figure 8.52. Note that the resonant frequency of the stub resonators is no longer \( \omega_0 \), since the resonator capacitor values have been modified by the \( \Delta C_n \). This implies that the length of the resonator is less than \( \lambda/4 \) at \( \omega_0 \), the filter center frequency. The transformation of the stub length to account for the change in capacitance is illustrated in Figure 8.53d. A short-circuited length of line with a shunt capacitor at its
8.8 Filters Using Coupled Resonators

FIGURE 8.53
Equivalent circuit for the bandpass filter of Figure 8.52. (a) A general bandpass filter circuit using shunt resonators with admittance inverters. (b) Replacement of admittance inverters with the circuit implementation of Figure 8.38d. (c) After combining shunt capacitor elements. (d) Change in resonant stub length caused by a shunt capacitor.
input has an input admittance of

\[ Y = Y_L + j\omega_0 C, \quad (8.139a) \]

where

\[ Y_L = -\frac{j\cot \beta \ell}{Z_0} . \]

If the capacitor is replaced with a short length, \( \Delta \ell \), of transmission line, the input admittance would be

\[ Y = \frac{1}{Z_0} \left( \frac{Y_L + j\frac{1}{Z_0} \tan \beta \Delta \ell}{1 + jY_L \tan \beta \Delta \ell} \right) \approx Y_L + j\frac{\beta \Delta \ell}{Z_0} . \quad (8.139b) \]

The last approximation follows for \( \beta \Delta \ell \ll 1 \), which is true in practice for filters of this type. Comparing (8.139b) with (8.139a) gives the change in stub length in terms of the capacitor value:

\[ \Delta \ell = \frac{Z_0\omega_0 C}{\beta} = \left( \frac{Z_0\omega_0 C}{2\pi} \right) \lambda . \quad (8.140) \]

Note that if \( C < 0 \), then \( \Delta \ell < 0 \), indicating a shortening of the stub length. Thus the overall stub length is given by

\[ \ell_n = \frac{\lambda}{4} + \left( \frac{Z_0\omega_0 \Delta C_n}{2\pi} \right) \lambda , \quad (8.141) \]

where \( \Delta C_n \) is defined in (8.138). The characteristic impedance of the stub resonators is \( Z_0 \).

Dielectric material properties play a critical role in the performance of ceramic resonator filters. Materials with high dielectric constants are required in order to provide miniaturization at the frequencies typically used for wireless applications. Losses must be low to provide resonators with high \( Q \), leading to low passband insertion loss and maximum attenuation in the stopbands. In addition, the dielectric constant must be stable with changes in temperature to avoid drifting of the filter passband over normal operating conditions. Most materials that are commonly used in dielectric resonator filters are ceramics, such as barium tetratitanate, zinc/stontium titanate, and various titanium oxide compounds. For example, a zinc/stontium titanate ceramic material has a dielectric constant of 36, a \( Q \) of 10,000 at 4 GHz, and a dielectric constant temperature coefficient of \(-7 \text{ ppm}/^\circ C\).

**EXAMPLE 8.10  CAPACITIVELY COUPLED SHUNT RESONATOR BANDPASS FILTER DESIGN**

Design a third-order bandpass filter with a 0.5 dB equal-ripple response using capacitively coupled short-circuited shunt stub resonators. The center frequency is 2.5 GHz, and the bandwidth is 10%. The impedance is 50 \( \Omega \). What is the resulting attenuation at 3.0 GHz?

**Solution**

We first calculate the attenuation at 3.0 GHz. Using (8.71) to convert 3.0 GHz to normalized low-pass form gives

\[ \omega \leftarrow \frac{1}{\Delta} \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) = \frac{1}{0.1} \left( \frac{3.0}{2.5} - \frac{2.5}{3.0} \right) = 3.667 . \]
Then, to use Figure 8.27a, the value on the horizontal axis is
\[
\left| \frac{\omega}{\omega_c} \right| - 1 = |-3.667| - 1 = 2.667,
\]
from which we find the attenuation as 35 dB.

Next we calculate the admittance inverter constants and coupling capacitor values using (8.136) and (8.137):

\[
\begin{array}{cccc}
 n & g_n & Z_0 J_{n-1,n} & C_{n-1,n} (\text{pF}) \\
 1 & 1.5963 & & C_{01} = 0.2896 \\
 2 & 1.0967 & Z_0 J_{12} = 0.0594 & C_{12} = 0.0756 \\
 3 & 1.5963 & Z_0 J_{23} = 0.0594 & C_{23} = 0.0756 \\
 4 & 1.0000 & Z_0 J_{34} = 0.2218 & C_{34} = 0.2896 \\
\end{array}
\]

Then we use (8.138), (8.140), and (8.141) to find the required resonator lengths:

\[
\begin{array}{cccc}
 n & \Delta C_n (\text{pF}) & \Delta \ell_n (\lambda) & \ell (\text{deg}) \\
 1 & -0.3652 & -0.04565 & 73.6 \\
 2 & -0.1512 & -0.0189 & 83.2 \\
 3 & -0.3652 & -0.04565 & 73.6 \\
\end{array}
\]

Note that the resonator lengths are slightly less than 90° (\(\lambda/4\)). The calculated amplitude response of this design is shown in Figure 8.54. The stopband rolloff at high frequencies is less than at lower frequencies, and the attenuation at 3 GHz is seen to be about 30 dB, while our calculated value for a canonical lumped-element bandpass filter was 35 dB.

**FIGURE 8.54** Amplitude response of the capacitively coupled shunt resonator bandpass filter of Example 8.10.
Figure 8.55 shows a wideband receiver downconverter module employing a variety of different filter types.

REFERENCES


PROBLEMS

8.1 Sketch the $k-\beta$ diagram for the infinite periodic structure shown below. Assume $Z_0 = 75 \, \Omega$, $d = 1.0 \, \text{cm}$, $k = k_0$, and $L_0 = 1.25 \, \text{nH}$. 