Biomechanics Part I

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Biomechanics?

A Branch of Bioengineering, Multidisciplinary field

- Cellular Biomechanics
- Musculoskeletal Biomechanics
- Sports Biomechanics
- Neuro-musculo Biomechanics
- Plant Biomechanics
- Ocular Biomechanics
- Cardiovascular Biomechanics
- Respiratory Biomechanics
- Hemodynamics
Why does biomechanics matter?

1. How do your bone “know” how big and strong to be so that they can support your weight and deal with the loads?
   - Growth of bone by mechanical stimuli
   - Mechanical stress as feedback signal for bone growth and remodeling

2. How do your arteries “know” how big to be so that they can deliver just the right amount of blood to their distal capillary beds?
   - Endothelial cells lining the inner arterial surface sense shear stress and send signal to cells deeper in the artery wall to control the remodeling of the artery

3. What about biomechanics in everyday life? How can we move?
   - The most obvious application of biomechanics is in locomotion. Muscles generate forces and movement is realized through joints.

4. What about biomechanics in the treatment of disease and dysfunction?
   - Biomechanics play obvious role in the design of implants that have a mechanical function such as artificial joints, dental implants, heart valves.

Biomechanics?

- Mechanics? \[ F = ma \]
  - The study of forces and their effect upon matter

- Biomechanics?
  - Application of mechanical principles to the study of living organisms
  - Cover broad range of topics including
    - Material and structural properties of biological materials
    - Biocontrol systems regulating metabolism or voluntary motion
    - Kinematics and kinetics of human movement
    - Biofluid mechanics in the cardiovascular and respiratory systems
    - Heat and mass transfer into biological tissues
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References


History (of biomechanics & mechanics)

- Aristotle (384–322BC)
  - Statics
- Leonardo da Vinci (1452–1519)
  - Ball-and-socket joint, force acting along the line of muscle
- Galileo Galilei (1564–1642)
  - Forces and motions, jumping and gait of horses and insects
- Harvey (1578–1657)
  - Basis for blood circulation, arteries and veins
- Giovanni Borelli (1564–1642)
  - Human and animal movement, muscle contraction, hearts and intestines
  - Borelli Award in American Society of Biomechanics
- Robert Hooke (1635–1703)
  - Hooke’s Law, stress and strain of elastic materials
- Leonard Euler (1707–1783)
  - Generalized Newton’s laws to continuum, pulse waves in arteries

from Borelli’s De Motu Animalium (1680) showing human motion resulting from the action of muscle pairs on bones, serving as levers, allowed to move at joins.

History (of mechanics, biomechanics)

- Thomas Young (1773–1828)
  - Young’s modulus, vibration, wave theory
- Ernst Weber (1795–1878)
  - Scientific study of human gait
- Jean Leonard Marie Poiseuille (1797–1869)
  - Improved measurement of blood pressure
  - Blood flow, Hagen–Poiseuille Law
- Julius Wolff (1836–1902)
  - Wolff’s law of bone-remodeling
- Yuan-Cheng Fung (1919–)
  - Father of modern biomechanics
  - Biomechanics: Mechanical properties of living tissues, 2nd, 1993
  - V.C. Mow & R. Huskies
  - Basic Orthopaedic Biomechanics and Mechanobiology, 3rd, 2005
4.2 Basic Mechanics

4.2.1 Vector Mathematics

magnitude & direction
\[ F = F_i \mathbf{i} + F_j \mathbf{j} \]
\[ |F| = \sqrt{F_i^2 + F_j^2} \text{ or called norm} \]
\[ \theta = \arctan \frac{F_j}{F_i} \]

Fig. 4.1 2-dimensional representation of vector F

magnitude \( |F| \) & unit vector \( \mathbf{e}_F \)
\[ |F| = |F| \mathbf{e}_F \]
\[ = 10 \text{ lb} \left( \frac{2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}}{\sqrt{2^2 + 6^2 + 4^2}} \right) \]
the unit vector: \( \mathbf{e}_F = \left( \frac{2\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}}{\sqrt{2^2 + 6^2 + 4^2}} \right) = 0.267\mathbf{i} + 0.802\mathbf{j} + 0.534\mathbf{k} \)
magnitude of the unit vector = 1
magnitude of \( F \): \[ |F| = \sqrt{2.67^2 + 8.02^2 + 5.34^2} = 10 \text{ lb} \]

Vecors are added by summing their components
\[ \mathbf{A} = A_i \mathbf{i} + A_j \mathbf{j} + A_k \mathbf{k} \]
\[ \mathbf{B} = B_i \mathbf{i} + B_j \mathbf{j} + B_k \mathbf{k} \]
\[ \mathbf{C} = \mathbf{A} + \mathbf{B} = (A_i + B_i) \mathbf{i} + (A_j + B_j) \mathbf{j} + (A_k + B_k) \mathbf{k} \]
### 4.2 Basic Mechanics

**Dot products** (inner products) → output: scalar

\[ \mathbf{A} \cdot \mathbf{B} = AB \cos \theta \]

For example:
\[
\mathbf{A} = 3 \mathbf{i} + 2 \mathbf{j} + \mathbf{k} \text{ m}
\]
\[
\mathbf{B} = -2 \mathbf{i} + 3 \mathbf{j} + 10 \mathbf{k} \text{ N}
\]
\[
\mathbf{A} \cdot \mathbf{B} = 3(-2) + 2(3) + 1(10) = 10 \text{ Nm}
\]

Physical interpretation of dot product:
\[ \mathbf{A} \cdot \mathbf{B} \] is the projection of \( \mathbf{A} \) onto \( \mathbf{B} \)
or the projection of \( \mathbf{B} \) onto \( \mathbf{A} \)

**Work** is defined as the force that acts in the same direction as the motion of a body.

The work \( W \) done by \( \mathbf{F} \) is given by:
\[ \mathbf{F} \cdot \mathbf{d} \equiv Fd \cos \theta \]

---

### 4.2 Basic Mechanics

**Cross products** (outer products) → output: another vector

- The moment of a force about a point (or axis) is a measure of its tendency to cause rotation.
- A vector \( \mathbf{F} \) acting in the x-y plane at a distance from the body’s coordinate center \( O \). The vector \( \mathbf{r} \) points from \( O \) to the line of action of \( \mathbf{F} \). The cross product \( \mathbf{r} \times \mathbf{F} \) is a vector that points in the z direction along the body’s axis of rotation.

The moment \( \mathbf{M} \) resulting from crossing \( \mathbf{r} \) into \( \mathbf{F} \) is:
\[ \mathbf{M} = M_x \mathbf{i} + M_y \mathbf{j} + M_z \mathbf{k} \]

\( M_x, M_y, \) and \( M_z \) cause rotation of the body.

For example:
\[
\mathbf{A} = 3 \mathbf{i} + 2 \mathbf{j} + \mathbf{k} \text{ m}, \mathbf{B} = -2 \mathbf{i} + 3 \mathbf{j} + 10 \mathbf{k} \text{ N}
\]
\[
\mathbf{A} \times \mathbf{B} = 3(-2) \mathbf{i} \times \mathbf{i} + 3(3) \mathbf{i} \times \mathbf{j} + 3(10) \mathbf{i} \times \mathbf{k} + 2(-2) \mathbf{j} \times \mathbf{i} + 2(3) \mathbf{j} \times \mathbf{j} + 2(10) \mathbf{j} \times \mathbf{k}
\]
\[
+ 1(-2) \mathbf{k} \times \mathbf{i} + 1(3) \mathbf{k} \times \mathbf{j} + 1(10) \mathbf{k} \times \mathbf{k}
\]

the magnitude \( |\mathbf{A} \times \mathbf{B}| = AB \sin \theta \)
\[
\mathbf{A} \times \mathbf{B} = 9 \mathbf{k} - 30 \mathbf{j} + 4 \mathbf{k} + 20 \mathbf{i} - 2 \mathbf{j} - 3 \mathbf{i} = 17 \mathbf{i} - 32 \mathbf{j} + 13 \mathbf{k} \text{ Nm}
\]
4.2 Basic Mechanics

4.2.2 Coordinate Transformations

Transformation between local (anatomical) coordinate and global coordinate

A measured in terms of the coordinate system XYZ,
in terms of the unit vectors I, J, K.

\[ A = A_x I + A_y J + A_z K \]

The unit vectors I, J, K can be written
in terms of i, j, k in the xyz system,

\[ I = \cos \theta_{xz} i + \cos \theta_{yz} j + \cos \theta_{zx} k \]
\[ J = \cos \theta_{xy} i + \cos \theta_{yz} j + \cos \theta_{yx} k \]
\[ K = \cos \theta_{xz} i + \cos \theta_{yz} j + \cos \theta_{zx} k \]

Where \( \cos \theta_{xz} \) is the angle between i and I, and similarly for other angles

Fig 4.5 Vector A, measured with respect to coordinate system XYZ is related to coordinate system xyz via the nine direction cosines of Eq. (4.20).

\[ i j k \rightarrow i' j' k' \]  \[ i' j' k' \rightarrow i'' j'' k'' \]  \[ i'' j'' k'' \rightarrow i''' j''' k''' \]

\[ i' = \cos \theta_x i - \sin \theta_x k \]
\[ j' = j \]
\[ k' = \sin \theta_x i + \cos \theta_x k \]

\[ i'' = \cos \theta_y i' + \sin \theta_y j' \]
\[ j'' = \cos \theta_y j' - \sin \theta_y k' \]
\[ k'' = -\sin \theta_y i' + \cos \theta_y k' \]

\[ i''' = \cos \theta_z i'' - \sin \theta_z k'' \]
\[ j''' = \cos \theta_z j'' - \sin \theta_z k'' \]
\[ k''' = k'' \]

\[ [i'''] = \begin{bmatrix} \cos \theta_x & \sin \theta_x & 0 \\ -\sin \theta_x & \cos \theta_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_y & \sin \theta_y \\ 0 & -\sin \theta_y & \cos \theta_y \end{bmatrix} \begin{bmatrix} \cos \theta_z & 0 & -\sin \theta_z \\ \sin \theta_z & 0 & \cos \theta_z \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix} \]

4.2.3 Euler angles: Relation of two orthogonal coordinates

Fig 4.5 The unprimed coordinate system xyz undergoes three rotations: about the y-axis (left), about the x-axis (middle), and about the z-axis (right), yielding the new triple-primed coordinate system x''''y''''z'''' for a y-x-z rotation sequence.
4.2 Basic Mechanics

4.2.3 Static Equilibrium

Newton’s equations of motion: \[ \sum F = 0 \quad \& \quad \sum M = 0 \]

→ Applied to biological systems

→ Free-body diagram of the body segment of interest with all externally applied loads and reaction forces at the supports

The unit vectors \( \mathbf{I}, \mathbf{J}, \mathbf{K} \) can be written in terms of \( \mathbf{i}, \mathbf{j}, \mathbf{k} \) in the \( xyz \) system,

\[ \mathbf{I} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k} \]
\[ \mathbf{J} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k} \]
\[ \mathbf{K} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k} \]

Where \( \cos \theta_x \) is the angle between \( \mathbf{i} \) and \( \mathbf{I} \), and similarly for other angles.

Example 4.4 a Russell’s traction rig used to apply an axial, tensile force to a fractures femur for immobilization. (a) What magnitude weight \( w \) must be suspended from the free end of the cable to maintain the leg in static equilibrium? (b) Compute average tensile force applied to the thigh under these conditions

Solution: cable tension \( T \) is constant (throughout the pulley)

\[ \sum F = 0 \]
\[ F_1 + F_2 + F_3 + F_{\text{femur}} - mg \mathbf{j} = 0 \]
\[ F_1 = -F_j = -T \mathbf{j} \]
\[ F_2 = (-F_2 \cos 30^\circ) \mathbf{i} + (F_2 \sin 30^\circ) \mathbf{j} \]
\[ = (-T \cos 30^\circ) \mathbf{i} + (T \sin 30^\circ) \mathbf{j} \]
\[ F_3 = (F_3 \cos 40^\circ) \mathbf{i} + (F_3 \sin 40^\circ) \mathbf{j} \]
\[ = (T \cos 40^\circ) \mathbf{i} + (T \sin 40^\circ) \mathbf{j} \]
\[ F_{\text{femur}} = (F_{\text{femur}} \cos 20^\circ) \mathbf{i} - (F_{\text{femur}} \sin 20^\circ) \mathbf{j} \]

weight of the foot and leg is 0.061 times body weight (Table 4.1)

\[ mg \mathbf{j} = (0.061)(150 \text{ lb}) = 9.2 \text{ lb} \]

summing the \( x \) & \( y \) components:

\[ \sum F_x = 0 : -T - T \cos 30^\circ + T \cos 40^\circ + F_{\text{femur}} \cos 20^\circ = 0 \]
\[ \sum F_y = 0 : T \sin 30^\circ + T \sin 40^\circ - F_{\text{femur}} \sin 20^\circ = 9.2 = 0 \]

Answer: \( T = 12.4 \text{ lb} \), \( F_{\text{femur}} = 14.5 \text{ lb} \)
Example 4.6 The force plate has 4 force sensors, one at each corner, that read the vertical forces \( F_1, F_2, F_3, \) and \( F_4. \) If the plate is square with side of length \( L \) and forces \( F_1\sim F_4 \) are unknown, write two expressions that will give \( x \) and \( y \) locations of the resultant force \( R \)

\[
\sum F_z = 0 : F_1 + F_2 + F_3 + F_4 - R = 0
\]

The force plate remains horizontal; hence the sum of the moments about the \( x \) and \( y \) axes must each be zero.

Taking moment about the \( x \) axis,

\[
\sum M_x = 0 : F_2l + F_3l - Ry = 0
\]

\[
\therefore y = \frac{(F_2 + F_3)l}{R}
\]

Taking moment about the \( y \) axis,

\[
\sum M_y = 0 : F_1l + F_4l - Rx = 0
\]

\[
\therefore x = \frac{(F_1 + F_4)l}{R}
\]

The coordinate \( x \) and \( y \) locate the resultant \( R \)

Fig 4.9 A square force plate with sides of length \( L \) is loaded with resultant force \( R \) and detects the vertical forces at each corner, \( F_1 \sim F_4 \)

4.2 Basic Mechanics

4.2.4 Anthropomorphic Mass moments of Inertia

A body’s mass \( \rightarrow \) resists linear motion

Its mass moment of inertia \( \rightarrow \) resist rotation

\[
I = \int \rho \ r^2 \ dm
\]

where \( m \) is the body mass, \( r \) is the moment arm to the axis of rotation.

\[
I = \rho \int \ r^2 \ dV
\]

for a body with constant density \( \rho \). Where \( V \) is the volume.

The radius of gyration \( k \) is the moment arm between the axis of rotation and a single point where all of the body's mass is concentrated. A body segment may be treated as a point mass with moment of inertia,

\[
I = mk^2
\]

Where \( m \) is the body segment mass.

The moment of inertia \( \text{w.r.t} \) a parallel axis \( I \) is related to the moment of inertia \( \text{w.r.t} \) the body's center of mass \( I_{cm} \) via the parallel axis theorem:

\[
I = I_{cm} + md^2
\]

Where \( d \) is the perpendicular distance b/w the two parallel axes.
Static Analysis of the Skeletal System

- Static analysis
  - Interest is the prediction of the forces (such as muscle forces, joint contact forces, passive soft tissue forces) that are internal to the musculoskeletal system when supporting some external loads
  - Equilibrium equations (3 eqs. in 2D, 6 eqs. in 3D)

For 3D
\[
\sum_j F_{ij} = 0 \\
\sum_j r_{ij} \times F_{ij} + \sum_k M_{ik} = 0
\]

where
- \(i\) : the link number
- \(j\) and \(k\) : force or moment on link \(i\)
- \(F_{ij}\) is force number \(j\) on segment number \(i\)
- \(r_{ij} \times F_{ij}\) : moment w.r.t. a reference point on segment \(i\)
- \(M_{ik}\) : a pure moment on link \(i\)

For 2D (planar motion)
\[
\sum_j (F_x)_{ij} = 0 \\
\sum_j (F_y)_{ij} = 0 \\
\sum_j (F_y d_{ij}) + \sum_k M_{ik} = 0
\]

Static Analysis of the Skeletal System

- Example: shoulder abduction to exercise strengthen the deltoids (삼각근). What are the forces required to sustain a weight?

Assumptions:
- The joint contact force acts through a known point, the center of the humeral head.
- Know the lines of actions of each muscle force (from anatomy)
- Ligaments do not limit the motions → ignored
- Reduce the 10 muscle forces and 2 components of the contact force → statically indeterminate problem (muscle redundancy problem)
**Static Analysis of the Skeletal System**

\[ x \text{ direction force } \sum_{i=1}^{10} F_{M_i} \cos(\theta_i) + F_{Cx} = 0 \]
\[ y \text{ direction force } \sum_{i=1}^{10} F_{M_i} \sin(\theta_i) + F_{Cy} - 60 - 30 = 0 \]
\[ \text{moment at shoulder } \sum_{i=1}^{10} F_{M_i} d_i - (60)(50) - (30)(30) = 0 \]

3 equations with 12 unknowns (10 muscle forces, 2 contact forces) → Indeterminate

When we assume that the only deltoid is active muscle → 1 muscle unknown

\[ F_M \cos \theta + F_{Jx} = 0 \]
\[ F_M \sin \theta + F_{Jy} = 90 \]
\[ F_M d = 3900 \]

\[ d = 3 \text{ [cm]}, \theta = 175° \]

**Static Analysis of the Skeletal System**

- Internal forces are equivalent to external forces
  \[ \sum (F_j)_\text{int} = - \sum (F_j)_\text{ext} \]
  \[ \sum (M_j)_\text{int} = - \sum (M_j)_\text{ext} \]

Internal forces:
in joint contact forces, muscle forces, ligament forces, and other soft tissue forces (unknown)

External forces due to:
gravity and contact with external surfaces (known or estimated)

Many unknowns than Eqns.
Indeterminate!
Muscle redundancy problem

- A specific set of muscle actions provides the desired action under the control of the central nervous system
Static Analysis of the Skeletal System

- We can lump the unknowns together and represent the net effects of all the internal forces acting across each joint as a resultant intersegmental force vector $R_i$ and a resultant intersegmental moment vector $T_i$

$$R_i = - \sum (F_i)_{ext}$$

$$T_i = - \sum (M_i)_{ext}$$

- Interested only in resultant forces and moment $\rightarrow$ solvable

- Three connected links (foot, shank, and thigh). 18 equilibrium equations and 18 unknown representing the resultants at the ankle, knee and hip.

- For some cases, contact forces and soft tissue forces are of interest $\rightarrow$ muscle redundancy problem

4.2 Basic Mechanics

4.2.5 Equations of Motion

Newton’s Equations of motion:
net force $F$ and the resulting translational motion as

$$F = ma$$

Where $a$ is the linear acceleration of the body’s center mass for translation.

For rotation,

$$M = I\alpha$$

Where $I\alpha$ is the body's angular momentum. $\alpha$ is the angular velocity.

The rate of change of a body’s angular momentum is equal to the net moment $M$ acting on the body.
4.3 Mechanic of Materials

Stress
- A measure of internal forces acting within deformable body.
- Average force per unit area
- Internal forces acting b/w particles across imaginary internal surfaces
- Reaction to external forces applied on the body
- Distributed continuously within the body
- Beyond certain limits of material strength, high stress cause permanent shape changes and failure

Axial stress

Shear stress

### Mechanics of Materials

Mechanics of materials may be used
- To quantify tissue deformation
- To study distributed orthopedic forces
- To predict performance of orthopedic prostheses and surgical corrections

Axial stress, \( \sigma \), is calculated by

\[
\sigma = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{500 \text{ N}}{(4.17 \times 10^{-3} \text{ m})(12 \times 10^{-3} \text{ m})} = 10 \times 10^6 \text{ [N / m}^2\text{]} = 10 \text{ MPa}
\]

The maximum shear stress, \( \tau_{\text{max}} \), occurs at a 45° angle to the applied load

\[
\tau_{\text{max}} = \frac{F_{45^\circ}}{A_{45^\circ}} = \frac{(500 \text{ N}) \cos 45^\circ}{(4.17 \times 10^{-3} \text{ m})(12 \times 10^{-3} \text{ m}) / \cos 45^\circ} = 5 \text{ MPa}
\]

Two points were punched 15 \( mm \) apart on the long axis of the plate.
After 500N applied, those marks are an additional 0.00075mm apart.
Strain \( \epsilon \) is calculated

\[
\epsilon = \frac{\Delta l}{l} = \frac{0.00075 \text{ mm}}{15 \text{ mm}} = 50 \times 10^{-6} \ ( = 50 \mu )
\]

Fig 4.10 Bone plate used to fix bone fractures, with applied axial load.
4.3 Mechanics of Materials

The elastic modulus (Young’s modulus), \( E \),
- a measure of material’s resistance to distortion by a tensile or compressive load.

For linearly elastic materials, \( E \) is a constant

\[
E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} = \frac{10 \times 10^9 \text{Pa}}{5 \times 10^{-6}} = 200 \times 10^9 \text{Pa} = 200 \text{GPa}
\]

Metal and plastic
- Linear elastic in limited ranges
- Yield stress: nonlinear, plastic, permanent deformation begins
- Ultimate stress: failure

Biomaterial
- More complex
- Viscoelastic (creep, stress relaxation, etc)

Table 4.2 Tensile Yield and Ultimate Stresses and Elastic Moduli (E) for Some Common Orthopedic Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Stress (MPa)</th>
<th>Ultimate Stress (MPa)</th>
<th>Elastic Modulus (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stainless Steels</td>
<td>700</td>
<td>850</td>
<td>180</td>
</tr>
<tr>
<td>Cobalt alloy</td>
<td>490</td>
<td>700</td>
<td>200</td>
</tr>
<tr>
<td>Titanium alloy</td>
<td>1100</td>
<td>1250</td>
<td>110</td>
</tr>
<tr>
<td><strong>Bone</strong></td>
<td><strong>85</strong></td>
<td><strong>120</strong></td>
<td><strong>18</strong></td>
</tr>
<tr>
<td>PMMA (fixtative)</td>
<td></td>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>UHMWPE (bearing)</td>
<td>14</td>
<td>27</td>
<td>1</td>
</tr>
<tr>
<td>Patellar ligament</td>
<td></td>
<td></td>
<td>58</td>
</tr>
</tbody>
</table>

* Ultrahigh molecular weight polyethylene (UHMWPE)
* Polymethyl methacrylate (PMMA)
Hysteresis
- Elastic properties vary between loading and unloading (Fig 4.12)
- The energy stored in the bone during loading is not equal to the energy released during unloading. The energy difference is greater as maximum load increases (curve A to B to C). The missing is dissipated as heat due to internal friction and damage to the material.

The Shear modulus
- The shear modulus (modulus of rigidity), G relates shear stress to shear strain
- The modulus of rigidity is related to the elastic modulus via Poisson’s ratio, \( \nu \), where
\[
\nu = \frac{E_{\text{transverse}}}{E_{\text{longitudinal}}}
\]
- For linear elastic material, E, G, and \( \nu \) are
\[
G = \frac{E}{2(1 + \nu)}
\]

Bone is an anisotropic material
- Anisotropic material properties depend on the direction of loading w.r.t the material axis. Isotropic material properties are independent of direction (e.g. metal)
- Human cortical bone is transversely isotropic
- Stronger & stiffer in longitudinal direction than transverse direction
- Stronger in compression than in tension and weakest when loaded transversely in tension.

<table>
<thead>
<tr>
<th>Average anisotropic elastic properties of human femoral cortical bone</th>
<th>Average anisotropic and asymmetric ultimate stress properties of human femoral cortical bone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal modulus, E (GPa)</td>
<td>Longitudinal (MPa)</td>
</tr>
<tr>
<td>17.0</td>
<td>Tension</td>
</tr>
<tr>
<td>Transverse modulus, E (GPa)</td>
<td>Compression</td>
</tr>
<tr>
<td>11.5</td>
<td>193</td>
</tr>
<tr>
<td>Shear modulus (GPa)</td>
<td>Transverse (MPa)</td>
</tr>
<tr>
<td>3.3</td>
<td>Tension</td>
</tr>
<tr>
<td>Longitudinal Poisson’s ratio</td>
<td>Compression</td>
</tr>
<tr>
<td>0.46</td>
<td>133</td>
</tr>
<tr>
<td>Transverse Poisson’s ratio</td>
<td>Shear (MPa)</td>
</tr>
<tr>
<td>0.58</td>
<td>68</td>
</tr>
</tbody>
</table>

Strength/modulus ratio (Comp. & tension)
Aluminum = 0.45 and 0.73
Cortical bone = 1.14 and 0.78 → high-performance material in compression

Reilly and Burstein (1975) *J Biomechanics*
Example 4.8

Orthopedic nail-plate used to fix an intertrochanteric fracture.

An external force of 400N at stance. The nail-plate is rectangular stainless steel with cross-sectional dimensions of 10mm width and 5mm height. What forces, moments, stresses, and strains will develop in this orthopedic device?

Sol> nail-plate should carry 400N

1. Axial and transverse components of force
\[ F_x = 400N \cos 20^\circ = 376N \]
\[ F_y = 400N \sin 20^\circ = 137N \]

2. The axial load produces compressive normal stress
\[ \sigma_x = \frac{F_x}{A} = \frac{376N}{(0.005m)(0.01m)} = 7.52 \text{MPa} \]

3. The maximum shear stress due to axial load is
\[ \tau_{\text{max}} = \frac{\sigma_x}{2} = 3.76 \text{MPa} \]

Fig 4.13 An intertrochanteric nail plate. (right-top) the free-body diagram of the upper section of this device,(right-below) the free-body diagram of a section of this beam cut at a distance x from the left-hand support.

Example 4.8

4. The axial strain
\[ E = \frac{\sigma}{\varepsilon} = \frac{F / A}{\Delta \ell / \ell} \]
\[ E = \frac{F / A}{E} = \frac{376N}{(180 \times 10^5 \text{Pa})(0.005m)(0.01m)} = 4.18 \times 10^{-6} \]

Thus axial deformation is \[ \Delta \ell_{\text{axial}} = \varepsilon \ell = 2.51 \times 10^{-6} \text{m} \rightarrow \text{negligible} \]

5. The transverse load causes the cantilever section to bend
\[ \delta y = \frac{F_x x^2}{6EI} (3L - x) \]
\[ \delta y_{\text{max}} = \frac{FL^3}{3EI} = \frac{137N (0.06)^3}{3(180 \times 10^5 \text{N/m}^2)(10.42 \times 10^{-9} \text{m}^4)} = 5.26 \times 10^{-4} = 0.526 \text{mm} \]

Where, \[ I = \frac{1}{12} bh^3 \]
\[ = \frac{1}{12} (10 \times 10^{-3} \text{m})(5 \times 10^{-3} \text{m}) = 10.42 \times 10^{-9} \text{m}^4 \]

Fig 4.13 (Left) A beam fixed on the left and subjected to a downward load on the right undergoes bending, with the top of the beam in tension and the bottom in compression. The position where tension changes to compression is denoted the neutral axis, located at c. (Right) A beam of rectangular cross section with width b and height h resists bending via the area moment of inertia.
4.4 Viscoelastic Properties

Viscoelasticity
- Exhibit both viscous and elastic characteristic when undergoing deformation.
- Viscous materials (e.g. honey) resist shear flow and strain
- Elastic materials strain instantaneously to stress
- Time-dependent strain
- Biomaterials such as bone, tendon, ligament, cartilage, intervertebral disc display viscoelasticity.
- For accurate analysis, how to model viscoelastic properties?
  1. Maxwell model
  2. Voigt model (Kelvin–Voight model)
  3. Standard linear solid model (Kelvin–body model)
  4. Elastic modulus with complex number–frequency dependent elastic modulus, stress relaxation, creep, and hysteresis exhibited by arteries

4.4 Viscoelastic Properties
1) Varying material properties to the rate of applied loading
- stiffness & strength increase with increasing rate of loading

2) Hysteresis
- Different loading vs. unloading curve → energy dissipation

3) Creep
- Continue to deform at constant stress

4) Stress relaxation
- Stress decrease at constant strain
4.4 Viscoelastic Properties

Linear viscoelastic model
- Hookean elastic spring + Newtonian viscous dashpot element

\[ \sigma = E \epsilon \quad \sigma = \mu \dot{\epsilon} \]

\[ E : \text{Young’s modulus [Nm}^{-2}] \]
\[ \mu : \text{Coefficient of viscosity [Nm}^{-1}\text{s}] \]
\[ \dot{\epsilon} = \frac{d \epsilon}{dt} : \text{strain rate} \]

\[ \mu \dot{\epsilon} + E \epsilon = E \mu \dot{\epsilon} \]
\[ \sigma = E \epsilon + \mu \dot{\epsilon} \]

A parallel combination of a spring and dashpot

\[ \dot{\sigma} + \frac{1}{\mu} \{E_1 + E_2\} \sigma = E_1 \dot{\epsilon} + \frac{E_1 E_2 \epsilon}{\mu} \]

How to choose the simplest model that adequately captures the experimentally observed behavior of material?

4.4 Viscoelastic Properties

Creep response: Subjected to a constant stress \( \sigma(t) = 1 \)

1) For Maxwell model \( \dot{\sigma} = 0 \)
   - Governing Differential Equation: \( \dot{\epsilon} = \frac{1}{\mu} \)
     \( \leftrightarrow \) From \( \mu \dot{\epsilon} + E \sigma = E \mu \dot{\epsilon} \)
     - Dashpot infinitely stiff at the instant of any load applied.

   Initial Condition: \( \epsilon(0) = \frac{\sigma(0)}{E} = \frac{1}{E} \)
   Solution: \( \epsilon(t) - \epsilon(0) = \int_0^t \frac{1}{\mu} \, dt = \frac{t}{\mu} \)
   \[ \epsilon(t) = \frac{1}{E} + \frac{1}{\mu} t \]
   \[ c(t)_M = \frac{1}{E} + \frac{1}{\mu} t : \text{creep function} \]

2) For Kelvin–Voigt model

\[ c(t)_{KV} = \frac{1}{E} (1 - e^{-t/\tau_K}) : \text{creep function} \]
\[ \tau_K = \frac{\mu}{E} : \text{relaxation time} \]
4.4 Viscoelastic Properties

\[ c(t)_M = \frac{1}{E} + \frac{1}{\mu} \]

\[ s(t)_M = E e^{-\frac{t}{\tau_E}} \]

Figure: The creep and stress-relaxation functions for Maxwell, Kelvin–Voigt, and standard linear solid models, in response to the stress and strain time histories.

4.5 Cartilage, Ligament, Tendon, and Muscle

4.5.1 Cartilage—연골

- The articulating surfaces of bones are covered by articular cartilage
- Strongly viscoelastic
- Ultimate compressive stress $\sim 5$MPa
- 20% Elastic matrix (collagen type II), 70% water, 6% proteoglycans, and other elements.
- 2~4mm thick in hip and knee
- No blood or nerve supply $\Rightarrow$ difficult to detect and repair damage.
- Can be modeled as a single-phase elastic continuum.
- Time-dependent behavior $\Rightarrow$ biphasic or triphasic material.
4.5.1 Articular Cartilage

1) Composition and Microstructure

- Avascular, no nerves
- Triphasic material composed of a porous matrix, water, and ions

(a) Three phases of cartilage: porous matrix, water, and charged ions
(b) The molecular structure of the proteoglycan monomer and aggregate

(1) The porous matrix
- Type II collagen (10~20%)
- Proteoglycans (6%): complex macromolecule that consist of a central long protein core, which has linkage of long polysaccharide chains. As large aggregating type (aggrecans) in articular cartilage
- Most aggrecan molecules bind together on a long chain. Chondroitin sulfate and keratan sulfate glycosaminoglycan

(2) Water
- 70~90% in wet weight
- Crucial contributor to the overall mechanical behavior. Most aggrecan molecules bind together on a long chain. Chondroitin sulfate and keratan sulfate glycosaminoglycan

(3) Ions
- Glycosaminoglycan chains contain many COOH and SO₄ groups, which in solution form COO⁻ and SO₄⁻ negative charged ions.
- These negative ions require positive ions such as Ca²⁺ and Na⁺ to maintain electroneutrality.
- This ionic phase of cartilage also affects its mechanical behavior.
4.5.1 Articular Cartilage

- Typical thickness: 2~4mm in most human joint.
- Four main zones
  1. Superficial tangential zone
     - Collagen fibers: tangent to the surface.
     - Collagen content highest, proteoglycan lowest.
  2. Middle zone (transitional zone)
     - 40~60 percent of the thickness
     - Isotropic area
  3. Deep zone (radial zone)
     - 30 percent of the thickness
     - Highest concentration of proteoglycan and lowest water content. Collagen fiber is thickest.
  4. Calcified cartilage zone
     - Combination of cartilage and mineral

Articular cartilage composition, biology, and mechanical properties through the thickness are determined by the developmental history and local mechanical conditions.

From Wong and Carter 2003
4.5.1 Articular Cartilage

2) Mechanical Behavior and Material Properties

- Anisotropic and heterogeneous

- Poroelastic solid (due to following two sources)
  1. Intrinsic: flow-independent behavior of the collagen–proteoglycan matrix.
  2. Flow of the interstitial fluid through the matrix and the resistance of the matrix to this flow.
    - Fluid flow dominate the dynamic, in vivo behavior of cartilage
    - The intrinsic properties determine the parameters that restrict such flow.
    - Ionic phase produces swelling in the tissue, which contributes to the resistance of fluid flow through the matrix.

- All three phases are intimately related in how they influence the material behavior of articular cartilage.

4.5.1 Articular Cartilage

- Tensile modulus is measured from the traditional uniaxial tensile test, which is performed at a constant rate of strain
  - Dynamic young’s modulus: 40–400MPa
  - depends on the depth and orientation of the specimen and strain rate
  - Flow of fluid occurs, not intrinsic property
  - For faster testing, tensile modulus increases with increasing strain rate.

![Stress-Strain Curve](image-url)
4.5.1 Articular Cartilage

- The intrinsic compressive properties of cartilage
  - Obtained by the confined compression creep test
  - During loading, fluid can escape only from the top of the specimen through a porous platen
  - A steady state is reached when all fluid flow stops
  - If the test is repeated at successively higher level of applied stress, the overall stress–strain envelope is approximately linear for strains below about 20 percent.

  - Aggregate molulus
  $H_A = \frac{\text{equilibrium stress}}{\text{equilibrium strain}}$
  $0.3\sim1.3\text{MPa}$

![Setup for the confined compression test of articular cartilage](image)

4.5.1 Articular Cartilage

- The tensile equilibrium modulus increases with increasing collagen content, collagen network.
- Shear modulus increases with increasing collagen content.
- Shear testing: to study intrinsic viscoelastic property of collagen–proteoglycan matrix behavior and the effect disease on collagen behavior
  - no change in volume, no fluid flow
  - the solid matrix is only slightly viscoelastic, but more viscoelastic than pure collagen: viscoelastic behavior of cartilage is due to fluid flow within the matrix.

![Proposed mechanism for cartilage resistance to shear loading, where collagen fibers are stretched](image)

![Positive correlation b/w dynamic shear modulus and collagen content](image)
4.5.1 Articular Cartilage

- Increasing collagen content has no effect on the compressive equilibrium aggregate modulus: Resistance to compression is provided mainly by the repulsive negative charges of the trapped proteoglycan molecules.
  - Proteoglycan molecules are (−) charged, and tend them to swell.
  - Collagen constrain this separation and swelling of the proteoglycan molecules → induce internal tensile stresses.
- Higher proteoglycan content → higher compressive aggregate modulus
- Higher water content (thus dilution in proteoglycan) → lower compressive aggregate modulus: degenerative cartilage, osteoarthritis cartilage

4.5.1 Articular Cartilage

3) Dynamic Loading

- The increased stiffness of cartilage under dynamic loading (viscoelasticity) is due to the flow of fluid through the proteoglycan–collagen matrix and the resistance to such flow offered by the porous elastic matrix.
- Darcy’s law: Fluid flow through a permeable porous material
  
  \[ Q = \frac{\kappa A \Delta P}{t} \]

  \( \kappa \): hydraulic permeability coefficient
  A: cross-sectional area
  \( \Delta P \): pressure differential
  t: thickness

- The hydraulic permeability coefficient K is a material property.
  - Healthy cartilage: \(10^{-15} \sim 10^{-16} \text{ m}^4/\text{Ns}\)
  - Low value of permeability → large pressure gradient are required to force fluid through the specimen.

  Ex) Typical cartilage thickness: 3mm (tibia), \(Q/A=10\mu\text{m}/\text{s}\)
  → pressure differential: 30MPa
4.5.1 Articular Cartilage

- Strain-dependent permeability effect: The permeability of articular cartilage decreases with increasing applied strain. If compressed and then the pressure gradient applied, permeability decreases due to both closing of the pores and an increase in the fixed charge density.
- The cartilage decreases its own permeability as it is loaded more to prevent further loss of fluid and increases its dynamic stiffness and thereby stopping itself from bottoming out.
- External dynamic loads are supported by a complex interaction of stress in the matrix and pressure in the fluid.
- Osteoarthritis → proteoglycan ↓, water content ↑
  - Degrade the intrinsic properties of the matrix
  - Permeability increases

![Permeability vs Compressive Strain](image)

4.5.1 Articular Cartilage

- Ironic phase
- Swelling: resisting external loads, caused by following mechanisms:
  1. Chemical expansion stress – mutually repulsive negative charges of the proteoglycan
  2. Donnan osmotic pressure – positive iron are attracted into the cartilage in order to attain electroneutrality. This results in an increase in ion concentration within the tissue. Thereby, fluid is attracted into the cartilage to reduce concentration gradient.
     - Fixed charge density for healthy cartilage is higher than that for osteoarthritis
- The total swelling pressure within cartilage is the sum of the chemical expansion stress and the Donnan osmotic pressure.
- The total swelling pressure is equilibrated against the elastic constraint of the stretched collagen fiber and an externally applied stress.
- Simple articular cartilage model – linearly elastic material with a modulus equal to a typical dynamic tangent modulus for a specified loading rate. (typically 10~20MPa)
4.5.1 Articular Cartilage

- Arthritis: deterioration of the joints
  - Osteoarthritis occurs when some combination of mechanical wear and biochemical degradation erodes the articular cartilage.
  - Localized effect and most common in the knee and hip since these joints bear the largest loads in the skeleton.
  - Damage to articular cartilage may be caused by normal wear and tear, trauma, or disease. For example, knee injury \( \Rightarrow \) osteoarthritis
  - Osteoarthritis may result from the deterioration of joint surfaces that occurs with long-term use.
  - Repeated insults may produce osteoarthritis (jackhammer)
  - Rheumatoid arthritis is caused by systematic inflammatory disease, affects multiple joints

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**Osteoarthrosis**

Healthy articular cartilage

Degenerated articular cartilage
**Radiographic evidence of Osteoarthrosis**

Cartilage thinning in the highly loaded regions follows as a result of increased frictional shear and wear.

![Normal Hip](image1) ![Early Osteoarthritis](image2)

*From Bullough 1992*

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**Thinning of Articular Cartilage**

Advance of the subchondral growth front

Surface wear and fibrillation

![Thinning of Articular Cartilage](image3)

*From Bullough 1992*

*From Harrison et al 1953*
Intermediate Osteoarthrosis of the Hip

- Loss of joint space
- Superior/lateral migration of the joint
- Growth of osteophytes (골증식체)

Late stage of Hip Osteoarthritis

- Osteophytes are prominent and threaten to “bridge” the joint at the joint margins
- In the joint contact areas there is increased cartilage turnover. The cartilage is trying to ossify but bone can’t form under conditions of high intermittent pressure—thus neochondrogenesis accompanies cartilage destruction

From Bullough 1992
Mechanical factors that promote osteoarthritis

- Physical damage to the superficial cartilage. For example, wear and fibrillation due to altered joint kinematics (joint laxity or ligament injury)
- Blunt impact causing microdamage at the cartilage/bone interface—shear stresses activate subchondral bone front
  - Cartilage loss with bone sclerosis
- Reduced joint loading—diminished hydrostatic pressure activates the subchondral growth front
  - Cartilage loss with bone osteopenia
- Other stress-related abnormalities—Osteochondral fractures, congenital or acquired deformities, obesity, physical activities

4.5 Cartilage, Ligament, Tendon, and Muscle

4.5.2 Ligaments and Tendons—인대와 힘줄

- Composed mainly of parallel bundles of collagen fibers
- Similar mechanical properties, Viscoelastic behaviors

- **Tendon**
  - Connect muscle to bone
  - Transmit large force during habitual activities
  - 60% water, By dry weight, 75~85% collagen type I, 1~3% elastin, 1~2% proteoglycan
  - Fibrils align with direction of the tendon, which is along the line of action

- **Ligament**
  - Connect bone to bone
  - Carry small force unless the joint is close to full range of motion
  - 60% water, By dry weight, 70~80% collagen type I, 1~15% elastin, 1~3% proteoglycan
  - More complex orientation of its fibrils than tendon
4.5.2 Ligaments and Tendons

Tendon: Collagen fibrils within the fascicle are aligned parallel to each other, but slightly crimped when unloaded.

Ligament: Fascicles do not have to be aligned with the overall orientation of the ligament→ lack of alignment distinguishes ligament from tendon.

Figure: Arrangement of collagen molecules into tendon, showing the various hierarchical structures.

4.5.2 Ligaments and Tendons

Mechanical Behavior

1. Toe region (due to crimped)
2. Linear region
3. Failure region
   - The initial nonlinear toe region is longer for ligament, since its collagen fibers are less aligned.

Nonlinear relation b/w tensile stress \((\sigma = \frac{F}{A_{\text{initial}}})\) and stretch \((\lambda = \frac{L}{L_{\text{initial}}})\)

Where, \(F=\)force, \(A=\)cross-sectional area, and \(L=\) length of the specimen.

\[
\sigma = a(e^{b\lambda} - 1)
\]

where \(a\) and \(b\) are found experimentally.

\(\Rightarrow\) tangent modulus-stress relationship \(\frac{d\sigma}{d\lambda} = b\sigma + c\)

where \(c = ab\), represents the tangent modulus as \(\sigma \to 0\).

- Tangent modulus increases linearly with increasing stress.

Fig 4.2 Typical Stress–strain curves
4.5.2 Ligaments and Tendons

- Mechanical and Structural Properties
  - Material properties of ligament and tendon strongly depend upon anatomic site (ligament type), age and strain rate

  - Material properties of Ligaments
    - Modulus: 30~500MPa
    - Ultimate stress: 4~45MPa
      - 20yrs, young 38MPa vs. 50~80 yrs, old 13MPa
    - Ultimate strain: 10~120%

  - Material properties of Tendons
    - Modulus: 60~2300MPa
    - Ultimate stress: 25~120MPa
    - Ultimate strain: 10~60%

4.5.2 Ligaments and Tendons

- Generally interested in structural properties, as organ
- Depend upon their size, geometry, sites and species
- In general knee ligaments are strongest in the body, ankle/elbow ligaments are weaker, spinal ligaments vary considerably

- Ankle Ligaments
  - Anterior & posterior fibulotalar ligament, the fiburocalcaneous ligament, tibiocalcaneal ligament

- Elbow Ligament
  - Anterior & posterior bundles of medial collateral ligament and the radial collateral ligament

- Spine
  - Anterior & posterior longitudinal ligament
4.5.2 Ligaments and Tendons

- Mechanical and Structural Properties [Continued]
  - **Knee Ligaments**
    - Anterior & posterior cruciate ligament, the lateral collateral ligament, medial collateral ligament
    - Failure to reproduce the correct force–deformation characteristic (thus kinematics) jeopardize the longevity of surgical reconstruction

![Diagram showing knee ligaments](image1)

- Different bundles take up different loads during in vivo functional loading

4.5.2 Ligaments and Tendons

- In Vitro and In Vivo Properties
  - In vivo behavior is different from in vitro (experiment) behavior
  - Slack region in tendon (maybe ligament, too?) unlikely exists in vivo (due to preload)
  - Depend upon their size, geometry, sites and species
  - Stiffness and Strength depends weakly on loading rate

![Graph showing load-time response](image2)

Fig. Load–time response to stretching of AM bundle of porcine ACL
Stress relaxation
4.5.2 Ligaments and Tendons

- Aging, Diseases, and Adaptation
  - Strength decreases substantially with age
  - Stiffness also decreases but not in vivo behavior is different from in vitro (experiment) behavior
  - Slack region in tendon (maybe ligament, too) unlikely exists in vivo (due to preload)
  - Damage to the ligament (due to sport injuries) → surgical treatment
    ACL reconstruction (using allograft, autograft): Successful reconstruction of correct tension in ligament, kinematics needed

4.5.3 Muscle Mechanics

- Muscles and Tendons
  - Muscle–tendon units
    - An active, excitable tissue that generates force by forming cross-bridge bonds between the interdigitating actin and myosin myofilaments
    - Convert chemical energy to mechanical work.
  - Total muscle force is the combination of
    - Active force generation
    - Visco–elastic behaviors of tendon
    - Passive muscle tissues
  - Force generated by the active element is a function of
    - Level of activation
    - Length of the muscle
    - Speed of contraction or lengthening
    → these functions can be represented mathematically but the model parameters are muscle specific.
4.5.3 Muscle Mechanics

• Classification of muscles
  1. Skeletal muscles: voluntary and striated
  2. Cardiac muscles: involuntary and striated
  3. Smooth (visceral) muscles: involuntary and not striated

• Functions of muscles
  – Activate, position, and stabilize the skeleton
    → Active contraction produces motion of one bone w.r.t another.
  – Co-contraction of agonist and antagonist muscles around a joint provides joint stability.
    (quadriceps vs. hamstrings)
    → Provide infinitely varying stiffnesses at the joint

4.5.3 Muscle Mechanics

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    • Level of activation
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    → these functions can be represented mathematically but the model parameters are muscle specific.
4.5.3 Muscle Mechanics

- Model of a skeletal muscle as an actuator includes four components:
  1. Contractile element that generates a force based on a neural stimulus
  2. Elastic element in series with the contractile element
  3. Parallel elastic element
  4. Parallel viscous element

- The passive elements represent the combined elastic and viscous effects of the muscle and the tendon

Fig 2.1 Contractile element, elastic element in series, parallel elastic element, parallel viscous element

4.5.3 Muscle Mechanics

- How to determine the line of action?
- Muscles have board attachments. Need several subunits to represent a muscle.
- The simplest approach
  - Assume that they act along a straight line (origin to insertion)
  - Almost all muscles don’t act along a straight line but useful results have been obtained with the use of straight line models of muscle
4.5.3 Muscle Mechanics

1) Composition and Microstructure
- Muscle cell < Muscle Filaments (Thin and Thick filament) < Myofibrils < Muscle fibers < Muscle
- Thin filament – protein Actin
- Thick filament – Myosin (head and tail)
- Cross-bridge rotates when ATP attaches to the myosin

- Sarcomere: thin & thick filaments arranged parallel, 2.6μm long, Z-line to Z-line
- Muscle relaxed: filaments overlap slightly → Muscle contracted: overlap more, sarcomere shorten

2) Mechanical Behavior and Active Force Generation
- Motor neuron stimulate a muscle by sending a short electrical potential to the muscle cell
- All-or-none principle: Above threshold, all muscle fibers in a motor unit contract fullest extent
- Strength of contraction depends on oxygen supply and ATP levels
- The magnitude of force is controlled by varying the number of motor neurons that are active and their frequency of activation

- Muscle response delay ~20ms
  1) Twitch response
  2) Unfused-tetanus: at low frequency
  3) Fused-tetanus: at high frequency, 60Hz
  Force-time response reaches a plateau with only a tiny ripple
  Twitch-tetanus ratio: 1.5~10

Response of muscle to an applied electrical stimulus.
4.5.3 Muscle Mechanics

3) Force-Length Behavior

- Passive and Active behavior
- Passive behavior (inactivated) similar to tendon/ligament
- When fully activated (all fibers stimulated at tetanus), the magnitude of force depends upon the length of the muscle

- Experiment: isometric (constant length) contraction, the length between the grip is changed. Force at that length is measured

\[ T = f(L) \]

![Fig 4.26 Tension-length experiment. x point- active, • point- passive](image)

4.5.3 Muscle Mechanics

- The active response – passive response = force produced exclusively by the contractile machinery of the muscle
- The maximum tension occurs at the resting length
  - Muscle length is too small, it cannot produce force when stimulated
  - Muscle length is too large, it cannot produce force from electrical stimulation (but large passive force)

\[ T = f(L/L_o) \]

![Fig 4.27 Tension-length curve, showing the active, passive, and developed tension responses. The length variable is normalized by the resting length \( L_o \)](image)
4.5.3 Muscle Mechanics

4) Force–Velocity Behavior

- The amount of force depends upon how fast the muscle is contracting
- For muscle shortening, max force at rest (isometric contraction)
- As the velocity of shortening increases, the force quickly falls off (Fig. 4.28)
- For muscle lengthening, maximum force is about 1.8 times the isometric contraction value. Plateau regardless of lengthening velocity.

- Max power (velocity x force) at a shortening velocity of 1/3 max velocity
- e.g. Bicycle gear

Fig 4.28 Force–velocity and power–velocity relations. Normalized by $v_{max}$ Max. power is produced at about 1/3 of the maximum shortening velocity.