5.1 Describe the characteristics of (1) an alloy, (2) pearlite, (3) austenite, (4) martensite, and (5) cementite.

(a) Alloy: composed of two or more elements, at least one element is a metal. The alloy may be a solid solution or it may form intermetallic compounds.

(b) Pearlite: a two-phase aggregate consisting of alternating lamellae of ferrite and cementite; the closer the pearlite spacing of lamellae, the harder the steel.

(c) Austenite: also called gamma iron, it has a fcc crystal structure which allows for a greater solubility of carbon in the crystal lattice. This structure also possesses a high ductility, which increases the steel’s formability.

(d) Martensite: forms by quenching austenite. It has a bct (body-centered tetragonal) structure, and the carbon atoms in interstitial positions impart high strength. It is hard and very brittle.

(e) Cementite: also known as iron-carbide (Fe₃C), it is a hard and brittle intermetallic phase.

5.2 What are the effects of mold materials on fluid flow and heat transfer?

The most important factor is the thermal conductivity of the mold material; the higher the conductivity, the higher the heat transfer and the greater the tendency for the fluid to solidify, hence possibly impeding the free flow of the molten metal. Also, the higher the cooling rate of the surfaces of the casting in contact with the mold, the smaller the grain size and hence the higher the strength. The type of surfaces developed in the preparation of mold materials may also be different. For example, sand-mold surfaces are likely be rougher than those of metal molds whose surfaces can be prepared to varying degrees of roughness, including the directions of roughness (lay).

5.3 How does the shape of graphite in cast iron affect its properties?

The shape of graphite in cast irons has the following basic forms:

(a) Flakes. Graphite flakes have sharp edges which act as stress raisers in tension. This shape makes cast iron low in tensile strength and ductility, but it still has high compressive strength. On the other hand, the flakes also act as vibration dampers, a characteristic important in damping of machine-tool bases and other structures.

(b) Nodules. Graphite can form nodules or spheroids when magnesium or cerium is
added to the melt. This form has increased ductility, strength, and shock resistance compared to flakes, but the damping ability is reduced.

(c) Clusters. Graphite clusters are much like nodules, except that they form from the breakdown of white cast iron upon annealing. Clusters have properties that are basically similar to flakes.

(d) Compacted flakes. These are short and thick flakes with rounded edges. This form has properties that are between nodular and flake graphite.

5.4 Explain the difference between short and long freezing ranges. How are they determined? Why are they important?

Freezing range is defined by Eq. (5.3) on p. 196 in terms of temperature difference. Referring to Fig. 5.6 on p. 197, note that once the phase diagram and the composition is known, we can determine the freezing range, $T_L - T_S$. As described in Section 5.3.2 on p. 196, the freezing range has an important influence on the formation and size of the mushy zone, and, consequently, affects structure-property relationships of the casting.

5.5 We know that pouring molten metal at a high rate into a mold has certain disadvantages. Are there any disadvantages to pouring it very slowly? Explain.

There are disadvantages to pouring metal slowly. Besides the additional time needed for mold filling, the liquid metal may solidify or partially solidify while still in the gating system or before completely filling the mold, resulting in an incomplete or partial casting. This can have extremely detrimental effects in a tree of parts, as in investment casting.

5.6 Why does porosity have detrimental effects on the mechanical properties of castings? Which physical properties are also affected adversely by porosity?

Pores are, in effect, internal discontinuities that are prone to cracking and crack propagation. Thus, the toughness of a material will decrease as a result of porosity. Furthermore, the presence of pores in a piece of metal under tension indicates that the material around the pores has to support a greater load than if no pores were present; thus, the strength is also lowered. Considering thermal and electrical conductivity, an internal defect such as a pore decreases both the thermal and electrical conductivity, noting that air is a very poor conductor.

5.7 A spoked hand wheel is to be cast in gray iron. In order to prevent hot tearing of the spokes, would you insulate the spokes or chill them? Explain.

Referring to Table 5.1 on p. 206, we note that, during solidification, gray iron undergoes an expansion of 2.5%. Although this fact may suggest that hot tearing cannot occur, consideration must also be given to significant contraction of the spokes during cooling. Since the hot-tearing tendency will be reduced as the strength increases, it would thus be advisable to chill the spokes to develop this strength.

5.8 Which of the following considerations are important for a riser to function properly? (1) Have a surface area larger than that of the part being cast. (2) Be kept open to atmospheric pressure. (3) Solidify first. Explain.

Both (1) and (3) would result in a situation contrary to a riser’s purpose. That is, if a riser solidifies first, it cannot feed the mold cavity. However, concerning (2), an open riser has some advantages over closed risers. Recognizing that open risers have the danger of solidifying first, they must be sized properly for proper function. But if the riser is correctly sized so that it remains a reservoir of molten metal to accommodate part shrinkage during solidification, an open riser helps exhaust gases from the mold during pouring, and can thereby eliminate some associated defects. A so-called blind riser that is not open to the atmosphere may cause pockets of air to be trapped, or increased dissolution of air into the metal, leading to defects in the cast part. For these reasons, the size and placement of risers is one of the most difficult challenges in designing molds.

5.9 Explain why the constant $C$ in Eq. (5.9) depends on mold material, metal properties, and temperature.
The constant $C$ takes into account various factors such as the thermal conductivity of the mold material and external temperature. For example, Zircon sand has a higher thermal conductivity than basic silica sand, and as a result, a casting in Zircon (of equal volume and surface area) will require less time to solidify than that cast in silica.

5.10 Explain why gray iron undergoes expansion, rather than contraction, during solidification.

As gray cast iron solidifies, a period of graphitization occurs during the final stages, which causes an expansion that counteracts the shrinkage of the metal. This results in an overall expansion.

5.11 How can you tell whether a cavity in a casting are due to porosity or to shrinkage?

Evidence of which type of porosity is present, i.e., gas or shrinkage, can be gained by studying the location and shape of the cavity. If the porosity is near the mold surface, core surface, or chaplet surface, it is most likely to be gas porosity. However, if the porosity occurs in an area considered to be a hot spot in the casting (see Fig. 5.37 on p. 249), it is most likely to be shrinkage porosity. Furthermore, gas porosity has smooth surfaces (much like the holes in Swiss cheese) and is often, though not always, generally spherical in shape. Shrinkage porosity has a more textured and jagged surface, and is generally irregular in shape.

5.12 Explain the reasons for hot tearing in castings.

Hot tearing is a result of tensile stresses that develop upon contraction during solidification in molds and cores if they are not sufficiently collapsible and/or do not allow movement under the resulting pressure during shrinkage.

5.13 Would you be concerned about the fact that a portion of an internal chill is left within the casting? What materials do you think chills should be made of, and why?

The fact that a part of the chill remains within the casting should be a consideration in the design of parts to be cast. The following factors are important:

(a) The material from which the chill is made should be compatible with the metal being cast (it should have approximately the same composition of the metal being poured).

(b) The chill must be clean, that is, without any lubricant or coating on the surface, because any gas evolved when the molten metal contacts the chill may not readily escape.

(c) The chill may not fuse with the casting, developing regions of weakness or stress concentration. If these factors are understood and provided for, the fact that a piece of the chill remains within the casting is generally of no significant concern.

5.14 Are external chills as effective as internal chills? Explain.

The effectiveness will depend on the location of the region to be chilled in the mold. If a region needs to be chilled (say, for example, to directionally solidify a casting), an external chill can be as effective as an internal chill. Often, however, chilling is required at some depth beneath the surface of a casting to be effective. For this condition an internal chill would be more effective.

5.15 Do you think early formation of dendrites in a mold can impede the free flow of molten metal into the mold? Explain.

Consider the solidification of an alloy with a very long freezing range. The mushy zone for this alloy will also be quite large (see Fig. 5.6). Since the mushy condition consists of interlacing dendrites surrounded by liquid, it is apparent that this condition will restrict fluid flow, as also confirmed in practice.

5.16 Is there any difference in the tendency for shrinkage void formation for metals with short freezing and long freezing ranges, respectively? Explain.

In an alloy with a long freezing range, the presence of a large mushy zone is more likely to occur, and thus the formation of microporosity. However, in an alloy with a short freezing range, the formation of gross shrinkage voids is more likely to occur.
5.17 It has long been observed by foundrymen that low pouring temperatures (that is, low superheat) promote equiaxed grains over columnar grains. Also, equiaxed grains become finer as the pouring temperature decreases. Explain the reasons for these phenomena.

Equiaxed grains are present in castings near the mold wall where rapid cooling and solidification take place by heat transfer through the relatively cool mold. With low pouring temperature, cooling to the solidification temperature is faster because there is less heat stored in the molten metal. With a high pouring temperature, cooling to the solidification temperature is slower, especially away from the mold wall. The mold still dissipates heat, but the metal remains molten for a longer period of time, thus producing columnar grains in the direction of heat conduction. As the pouring temperature is decreased, equiaxed grains should become finer because the cooling is more rapid and large grains do not have time to form from the molten metal.

5.18 What are the reasons for the large variety of casting processes that have been developed over the years?

By the student. There are several acceptable answers depending on the interpretation of the problem by the student. Students may approach this as processes that are application driven, material driven, or economics driven. For example, while investment casting is more expensive than sand casting, closer dimensional tolerances and better surface finish are possible. Thus, for certain parts such as barrels for handguns, investment casting is preferable. Consider also the differences between the hot- and cold-chamber permanent-mold casting operations.

5.19 Why can blind risers be smaller than open-top risers?

Risers are used as reservoirs for a casting in regions where shrinkage is expected to occur, i.e., areas which are the last to solidify. Thus, risers must be made large enough to ensure that they are the last to solidify. If a riser solidifies before the cavity it is to feed, it is useless. As a result, an open riser in contact with air must be larger to ensure that it will not solidify first. A blind riser is less prone to this phenomenon, as it is in contact with the mold on all surfaces; thus a blind riser may be made smaller.

5.20 Would you recommend preheating the molds in permanent-mold casting? Also, would you remove the casting soon after it has solidified? Explain.

Preheating the mold in permanent-mold casting is advisable in order to reduce the chilling effect of the metal mold which could lead to low metal fluidity and the problems that accompany this condition. Also, the molds are heated to reduce thermal damage which may result from repeated contact with the molten metal. Considering casting removal, the casting should be allowed to cool in the mold until there is no danger of distortion or developing defects during shakeout. While this may be a very short period of time for small castings, large castings may require an hour or more.

5.21 In a sand-casting operation, what factors determine the time at which you would remove the casting from the mold?

This question is an important one for any casting operation, not just sand casting, because a decrease in production time will result in a decrease in product cost. Therefore, a casting ideally should be removed at the earliest possible time. Factors which affect time are the thermal conductivity of the mold-material and of the cast metal, the thickness and the overall size of the casting, and the temperature at which the metal is being poured.

5.22 Explain why the strength-to-weight ratio of diecast parts increases with decreasing wall thickness.

Because the metal die acts as a heat sink for the molten metal, the metal chills rapidly, developing a fine-grain hard skin with higher strength. As a result, the strength-to-weight ratio of diecast parts increases with decreasing wall thickness.

5.23 We note that the ductility of some cast alloys is very low (see Fig. 5.13). Do you think this should be a significant concern in engineering applications of castings? Explain.
The low ductility of some cast alloys should certainly be taken into consideration in the engineering application of the casting. The low ductility will:

(a) affect properties, such as toughness and fatigue,
(b) have a significant influence on further processing and finishing of the casting, i.e., machining processes, such as milling, drilling, and tapping, and
(c) possibly affect tribological behavior.

It should be noted that many engineering applications do not require high ductility; for example, when stresses are sufficiently small to ensure the material remains elastic and where impact loads do not occur.

5.24 The modulus of elasticity of gray iron varies significantly with its type, such as the ASTM class. Explain why.

Because the shape, size, and distribution of the second-phase (i.e., the graphite flakes) vary greatly for gray cast irons, there is a large corresponding variation of properties attainable. The elastic modulus, for example, is one property which is affected by this factor.

5.25 List and explain the considerations involved in selecting pattern materials.

Pattern materials have a number of important material requirements. Often, they are machined, thus good machinability is a requirement. The material should be sufficiently stiff to allow good shape development. The material must have sufficient wear and corrosion resistance so that the pattern has a reasonable life. The economics of the operation is affected also by material cost.

5.26 Why is the investment-casting process capable of producing fine surface detail on castings?

The surface detail of the casting depends on the quality of the pattern surface. In investment casting, for example, the pattern is made of wax or a thermoplastic poured or injected into a metal die with good surface finish. Consequently, surface detail of the casting is very good and can be controlled. Furthermore, the coating on the pattern (which then becomes the mold) consists of very fine silica, thus contributing to the fine surface detail of the cast product.

5.27 Explain why a casting may have a slightly different shape than the pattern used to make the mold.

After solidification, shrinkage continues until the casting cools to room temperature. Also, due to surface tension, the solidifying metal will, when surface tension is high enough, not fully conform to sharp corners and other intricate surface features. Thus, the cast shape will generally be slightly different from that of the pattern used.

5.28 Explain why squeeze casting produces parts with better mechanical properties, dimensional accuracy, and surface finish than expendable-mold processes.

The squeeze-casting process consists of a combination of casting and forging. The pressure applied to the molten metal by the punch, or upper die, keeps the entrapped gases in solution, and thus porosity is generally not found in these products. Also, the rapid heat transfer results in a fine microstructure with good mechanical properties. Due to the applied pressure and the type of die used, i.e., metal, good dimensional accuracy and surface finish are typically found in squeeze-cast parts.

5.29 Why are steels more difficult to cast than cast irons?

The primary reason steels are more difficult to cast than cast irons is that they melt at a higher temperature. The high temperatures complicate mold material selection, preparation, and techniques involved for heating and pouring of the metal.

5.30 What would you recommend to improve the surface finish in expendable-mold casting processes?

One method of improving the surface finish of castings is to use what is known as a facing sand, such as Zircon. This is a sand having better properties (such as permeability and surface finish) than bulk sand, but is generally more expensive. Thus, facing sand is used as a first
layer against the pattern, with the rest of the mold being made of less expensive (silica) sand.

5.31 You have seen that even though die casting produces thin parts, there is a limit to the minimum thickness. Why can’t even thinner parts be made by this process?

Because of the high thermal conductivity that metal dies exhibit, there is a limiting thickness below which the molten metal will solidify prematurely before filling the mold cavity. Also, the finite viscosities of the molten metal (which increases as it begins to cool) will require higher pressures to force the metal into the narrow passages of the die cavities.

5.32 What differences, if any, would you expect in the properties of castings made by permanent-mold vs. sand-casting methods?

As described in the text, permanent-mold castings generally possess better surface finish, closer tolerances, more uniform mechanical properties, and more sound thin-walled sections than sand castings. However, sand castings generally can have more intricate shapes, larger overall size, and lower in cost (depending upon the alloy) than permanent-mold castings.

5.33 Which of the casting processes would be suitable for making small toys in large numbers? Explain.

This is an open-ended problem, and the students should give a rationale for their choice. Refer also to Table 5.2 and note that die casting is one of the best processes for this application. The student should refer to the application requiring large production runs, so that tooling cost per casting can be low, the sizes possible in die casting are suitable for such toys, and the dimensional tolerances and surface finish are acceptable.

5.34 Why are allowances provided for in making patterns? What do they depend on?

Shrinkage allowances on patterns are corrections for the shrinkage that occurs upon solidification of the casting and its subsequent contraction while cooling to room temperature. The allowance will therefore depend on the amount of contraction an alloy undergoes.

5.35 Explain the difference in the importance of drafts in green-sand casting vs. permanent-mold casting.

Draft is provided to allow the removal of the pattern without damaging the mold. If the mold material is sand and has no draft, the mold cavity is likely to be damaged upon pattern removal, due to the low strength of the sand mold. However, a die made of high-strength steel, which is typical for permanent-mold castings, is not at all likely to be damaged during the removal of the part; thus smaller draft angles can be employed.

5.36 Make a list of the mold and die materials used in the casting processes described in this chapter. Under each type of material, list the casting processes that are used, and explain why these processes are suitable for that particular mold or die material.

This is an open-ended problem, and students should be encouraged to develop an answer based on the contents of this chapter. An example of an acceptable answer would, in a brief form, be:

- Sand: Used because of its ability to resist very high temperatures, availability, and low cost. Used for sand, shell, expanded-pattern, investment, and ceramic-mold casting processes.
- Metal: Such as steel or iron. Results in excellent surface finish and good dimensional accuracy. Used for die, slush, pressure, centrifugal, and squeeze-casting processes.
- Graphite: Used for conditions similar to those for metal molds; however, lower pressures should be employed for this material. Used mainly in pressure- and centrifugal-casting.
- Plaster of paris: Used in plaster-mold casting for the production of relatively small components, such as fittings and valves.

5.37 Explain why carbon is so effective in imparting strength to iron in the form of steel.

Carbon has an atomic radius that is about 57% of the iron atom, thus it occupies an interstitial position in the iron unit cell (see Figs. 3.2 on
p. 84 and 3.9 on p. 90). However, because its radius is greater than that of the largest hole between the Fe atoms, it strains the lattice, thus interfering with dislocation movement and leading to strain hardening. Also, the size of the carbon atom allows it to have a high solubility in the fcc high-temperature phase of iron (austenite). At low temperatures, the structure is bcc and has a very low solubility of carbon atoms. On quenching, the austenitic structure transforms to body-centered tetragonal (bct) martensite, which produces high distortion in the crystal lattice. Because it is higher, the strength increase is more than by other element additions.

5.38 Describe the engineering significance of the existence of a eutectic point in phase diagrams.

The eutectic point corresponds to a composition of an alloy that has a lowest melting temperature for that alloy system. The low melting temperature associated with a eutectic point can, for example, help in controlling thermal damage to parts during joining, as is done in soldering. (See Section 12.13.3 starting on p. 776).

5.39 Explain the difference between hardness and hardenability.

Hardness represents the material’s resistance to plastic deformation when indented (see Section 2.6 starting on p. 51), while hardenability is the material’s capability to be hardened by heat treatment. (See also Section 5.11.1 starting on p. 236).

5.40 Explain why it may be desirable or necessary for castings to be subjected to various heat treatments.

The morphology of grains in an as-cast structure may not be desirable for commercial applications. Thus, heat treatments, such as quenching and tempering (among others), are carried out to optimize the grain structure of castings. In this manner, the mechanical properties can be controlled and enhanced.

5.41 Describe the differences between case hardening and through hardening insofar as engineering applications are concerned.

Case hardening is a treatment that hardens only the surface layer of the part (see Table 5.7 on p. 242). The bulk retains its toughness, which allows for blunting of surface cracks as they propagate inward. Case hardening generally induces compressive residual stresses on the surface, thus retarding fatigue failure. Through hardened parts have a high hardness across the whole part; consequently, a crack could propagate easily through the cross section of the part, causing major failure.

5.42 Type metal is a bismuth alloy used to cast type for printing. Explain why bismuth is ideal for this process.

When one considers the use of type or for precision castings such as mechanical typewriter impressions, one realizes that the type tool must have extremely high precision and smooth surfaces. A die casting using most metals would have shrinkage that would result in the distortion of the type, or even the metal shrinking away from the mold wall. Since bismuth expands during solidification, the molten metal can actually expand to fill molds fully, thereby ensuring accurate casting and repeatable typefaces.

5.43 Do you expect to see larger solidification shrinkage for a material with a bcc crystal structure or fcc? Explain.

The greater shrinkage would be expected from the material with the greater packing efficiency or atomic packing factor (APF) in a solid state. Since the APF for fcc is 0.74 and for bcc it is 0.68, one would expect a larger shrinkage for a material with a fcc structure. This can also been seen from Fig. 3.2 on p. 84. Note, however, that for an alloy, the answer is not as simple, since it must be determined if the alloying element can fit into interstitial positions or serves as a substitutional element.

5.44 Describe the drawbacks to having a riser that is (a) too large, or (b) too small.

The main drawbacks to having a riser too large are: the material in the riser is eventually scrapped and has to be recycled; the riser has to be cut off, and a larger riser will cost more
to machine; an excessively large riser slows solidification; the riser may interfere with solidification elsewhere in the casting; the extra metal may cause buoyancy forces sufficient to separate the mold halves, unless they are properly weighted or clamped. The drawbacks to having too small a riser are mainly associated with defects in the casting, either due to insufficient feeding of liquid to compensate for solidification shrinkage, or shrinkage pores because the solidification front is not uniform.

5.45 If you were to incorporate lettering on a sand casting, would you make the letters protrude from the surface or recess into the surface? What if the part were to be made by investment casting?

In sand casting, where a pattern must be prepared and used, it is easier to produce letters and numbers by machining them into the surface of a pattern; thus the pattern will have recessed letters. The sand mold will then have protruding letters, as long as the pattern is faithfully reproduced. The final part will then have recessed letters.

In investment casting, the patterns are produced through injection molding. It is easier to include recessed lettering in the injection molding die (instead of machining protruding letters). Thus, the mold will have recessed letters and the pattern will have protruding letters. Since the pattern is a replica of the final part, the part will also have protruding letters.

In summary, it is generally easier to produce recessed letters in sand castings and protruding letters in investment casting.

5.46 List and briefly explain the three mechanisms by which metals shrink during casting.

Metals shrink by:

(a) Thermal contraction in the liquid phase from superheat temperature to solidification temperature,

(b) Solidification shrinkage, and

(c) Thermal contraction in the solid phase from the solidification temperature to room temperature.

5.47 Explain the significance of the “tree” in investment casting.

The tree is important because it allows simultaneous casting of several parts. Since significant labor is involved in the production of each mold, this strategy of increasing the number of parts that are poured per mold is critical to the economics of investment casting.

5.48 Sketch the microstructure you would expect for a slab cast through (a) continuous casting, (b) strip casting, and (c) melt spinning.

The microstructures are as follows:

Note that the continuous cast structure shows the columnar grains growing away from the mold wall. The strip-cast metal has been hot rolled immediately after solidification, and is shown as quenched, prior it is annealed to obtain an equiaxed structure. The melt-spun structure solidifies so rapidly that there are no clear grains (an amorphous metal).

5.49 The general design recommendations for a well in sand casting are that (a) its diameter should be twice the sprue exit diameter, and (b) the depth should be approximately twice the depth of the runner. Explain the consequences of deviating from these rules.

Refer to Figure 5.10 for terminology used in this problem.

(a) Regarding this rule, if the well diameter is much larger than twice the exit diameter, liquid will not fill the well and aspiration of the molten metal may result. On the other hand, if the diameter is small compared to the sprue exit diameter, and recognizing that wells are generally not tapered, then there is a fear of aspiration within the well (see the discussion of sprue profile in Section 5.4 starting on p. 199.)
(b) If the well is not deeper than the runner, then turbulent metal first splashed into the well is immediately fed into the casting, leading to aspiration and associated defects. If the well is much deeper, then the metal remains in the well and can solidify prematurely.

5.50 Describe the characteristics of thixocasting and rheocasting.

Thixocasting and rheocasting involve casting operations where the alloy is in the slushy stage. Often, ultrasonic vibrations will be used to ensure that dendrites remain in solution, so that the metal is a slurry of molten continuous phase and suspended particles. In such casting operations, the molten metal has a lower superheat and, therefore, requires less cycle time, and shrinkage defects and porosity can be decreased. This is further described in Section 5.10.6 starting on p. 233.

5.51 Sketch the temperature profile you would expect for (a) continuous casting of a billet, (b) sand casting of a cube, (c) centrifugal casting of a pipe.

This would be an interesting finite-element assignment if such software is made available to the students. Consider continuous casting. The liquid portion has essentially a constant temperature, as there is significant stirring of the liquid through the continuous addition of molten metal. The die walls extract heat, and the coolant spray at the die exterior removes heat even more aggressively. Thus, a sketch of the isotherms in continuous casting would be as follows:

The cube and the pipe are left to be completed by the student.

5.52 What are the benefits and drawbacks to having a pouring temperature that is much higher than the metal’s melting temperature? What are the advantages and disadvantages in having the pouring temperature remain close to the melting temperature?

If the pouring temperature is much higher than that of the mold temperature, there is less danger that the metal will solidify in the mold, and it is likely that even intricate molds can be fully filled. This situation makes runners, gates, wells, etc., easier to design because their cross sections are less critical for complete mold filling. The main drawback is that there is an increased likelihood of shrinkage pores, cold shuts, and other defects associated with shrinkage. Also there is an increased likelihood of entrained air since the viscosity of the metal will be lower at the higher pouring temperature. If the pouring temperature is close to the melting temperature, there will be less likelihood of shrinkage porosity and entrained air. However, there is the danger of the molten metal solidifying in a runner before the mold cavity is completely filled; this may be overcome with higher injection pressures, but clearly has a cost implication.

5.53 What are the benefits and drawbacks to heating the mold in investment casting before pouring in the molten metal?

Heating the mold in investment casting is advisable in order to reduce the chilling effect of the mold, which otherwise could lead to low metal fluidity and the problems that accompany this condition. Molds are usually preheated to some extent. However, excessive heating will compromise the strength of the mold, resulting in erosion and associated defects.

5.54 Can a chaplet also act as a chill? Explain.

A chaplet is used to position a core. It has a geometry that can either rest against a mold face or it can be inserted into a mold face. If the chaplet is a thermally conductive material, it can also serve as a chill.
5.55 Rank the casting processes described in this chapter in terms of their solidification rate. For example, which processes extract heat the fastest from a given volume of metal and which is the slowest?

There is, as expected, some overlap between the various processes, and the rate of heat transfer can be modified whenever desired. However, a general ranking in terms of rate of heat extraction is as follows: Die casting (cold chamber), squeeze casting, centrifugal casting, slush casting, as it is being cooled to the following temperatures: (1) 750 °C (2) solidus temperature, (3) 726 °C, (4) the major phase at 1673 K, and (5) the ratio of solid to liquid at 1673 K.

We estimate the following quantities from Fig. 5.3 on p. 192: (1) The liquidus temperature is 1673 K. (2) The solidus temperature is 1645 K. (3) At 1645 K, the alloy is still all liquid, thus the nickel concentration is 80%. (4) The major phase at 1673 K is liquid, with no solids present since the alloy is not below the liquidus temperature. (5) The ratio is zero, since no solid is present.

5.57 Referring to Fig. 5.3, estimate the following quantities for a 20% Cu-80% Ni alloy: (1) liquidus temperature, (2) solidus temperature, (3) percentage of nickel in the liquid at 1673 K, (4) the major phase at 1673 K, and (5) the ratio of solid to liquid at 1673 K.

We estimate the following quantities from Fig. 5.6 on p. 197: (a) At 726 °C, the alloy is in the two-phase gamma-alpha field, and the percent gamma is 100% (10 kg), and alpha is 0%. (b) At 1001 K, the alloy is in the single-phase austenite (gamma) region, thus the percentage gamma is 100% (10 kg), and alpha is 0%. (c) At 999 K, the alloy is in the two-phase alpha and Fe3C field. No gamma phase is present. Again the lever rule is used to find the amount of alpha present:

\[ %\alpha = \left( \frac{x_\gamma - x_\alpha}{x_\gamma - x_o} \right) \times 100\% \]

\[ = \left( \frac{0.77 - 0.06}{0.77 - 0.022} \right) \times 100\% \]

\[ = 23\% \text{ or } 2.3 \text{ kg} \]

\[ %\gamma = \left( \frac{x_\alpha - x_o}{x_\gamma - x_o} \right) \times 100\% \]

\[ = \left( \frac{0.60 - 0.022}{0.77 - 0.022} \right) \times 100\% \]

\[ = 77\% \text{ or } 7.7 \text{ kg} \]

(c) At 999 K, the alloy is in the two-phase alpha and Fe3C field. No gamma phase is present. Again the lever rule is used to find the amount of alpha present:

\[ %\alpha = \left( \frac{6.67 - 0.60}{6.67 - 0.022} \right) \times 100\% = 91\% \text{ or } 9.1 \text{ kg} \]

5.58 Determine the amount of gamma and alpha phases (see Fig. 5.4b) in a 10 kg, AISI 1060 steel casting as it is being cooled to the following temperatures: (1) 1023 K, (2) 1001 K, and (3) 999 K.

We determine the following quantities from Fig. 5.6 on p. 197: (a) At 1023 K, the alloy is just in the single-phase austenite (gamma) region, thus the percentage gamma is 100% (10 kg), and alpha is 0%. (b) At 1001 K, the alloy is in the two-phase gamma-alpha field, and the weight percentages of each is found by the lever rule (see Example 5.1):

\[ %\alpha = \left( \frac{x_\gamma - x_o}{x_\gamma - x_\alpha} \right) \times 100\% \]

\[ = \left( \frac{0.77 - 0.06}{0.77 - 0.022} \right) \times 100\% \]

\[ = 23\% \text{ or } 2.3 \text{ kg} \]

5.59 A round casting is 0.3 m in diameter and 0.5 m in length. Another casting of the same metal is elliptical in cross section, with a major-to-minor axis ratio of 3, and has the same length and cross-sectional area as the round casting. Both pieces are cast under the same conditions. What is the difference in the solidification times of the two castings?
For the same length and cross-sectional area (thus the same volume), and the same casting conditions, the same \( C \) value in Eq. (5.11) on p. 205 should be applicable. The surface area and volume of the round casting is

\[
A_{\text{round}} = 2\pi rl + 2\pi r^2 = 0.613 \text{ m}^2
\]
\[
V_{\text{round}} = \pi r^2l = 0.2277 \text{ cm}^2
\]

Since the cross-sectional area of the ellipse is the same as that for the cylinder, and it has a major and minor diameter of \( a \) and \( b \), respectively, where \( a = 3b \), then

\[
\pi ab = \pi r^2
\]
\[
3b^2 = r^2 \quad \Rightarrow \quad b = \sqrt{\frac{(0.15)^2}{3}}
\]

or \( b = 0.0866 \text{ m} \), so that \( a = 0.260 \text{ m} \). The surface area of the ellipse-based part is (see a basic geometry text for the area equation derivations):

\[
A_{\text{ellipse}} = 2\pi ab + 2\pi \sqrt{a^2 + b^2}l = 1.002 \text{ m}^2
\]

The volume is still 0.089662 cm\(^2\). According to Eq. (5.11) on p. 205, we thus have

\[
\frac{T_{\text{round}}}{T_{\text{ellipse}}} = \left(\frac{V}{A_{\text{round}}}\right)^2 = \left(\frac{A_{\text{ellipse}}}{A_{\text{round}}}\right)^2 = 2.67
\]

5.60 Derive Eq. (5.7).

We note that Eq. (5.5) on p. 200 gives a relationship between height, \( h \), and velocity, \( v \), and Eq. (5.6) on p. 201 gives a relationship between height, \( h \), and cross-sectional area, \( A \). With the reference plate at the top of the pouring basin (and denoted as subscript 0), the sprue top is denoted as 1, and the bottom as 2. Note that \( h_2 \) is numerically greater than \( h_1 \). At the top of the sprue we have \( v_0 = 0 \) and \( h_0 = 0 \). As a first approximation, assume that the pressures \( p_o, p_1 \) and \( p_2 \) are equal and that the frictional loss \( f \) is negligible. Thus, from Eq. (5.5) we have

\[
h_o + \frac{p_o}{\rho g} + \frac{v_o^2}{2g} = h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + f
\]

or, solving for \( v_1 \),

\[
0 = h_1 + \frac{v_1^2}{2g} \quad \Rightarrow \quad v_1 = \sqrt{2gh_1}
\]

Similarly,

\[
h_o + \frac{p_o}{\rho g} + \frac{v_o^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + f
\]

or

\[
v_2 = \sqrt{2gh_2}
\]

Substituting these results into the continuity equation given by Eq. (5.6), we have

\[
A_1v_1 = A_2v_2
\]
\[
A_1\sqrt{2gh_1} = A_2\sqrt{2gh_2}
\]
\[
\frac{A_1}{A_2} = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}}
\]

which is the desired relationship.

5.61 Two halves of a mold (cope and drag) are weighted down to keep them from separating due to the pressure exerted by the molten metal (buoyancy). Consider a solid, spherical steel casting, 22.86 cm in diameter, that is being produced by sand casting. Each flask (see Fig. 5.10) is 50.8 cm by 50.8 cm and 38.1 cm deep. The parting line is at the middle of the part. Estimate the clamping force required. Assume that the molten metal has a density of 8009.2317 kg/m\(^3\) and that the sand has a density of 1601.8463 kg/m\(^3\).

The force exerted by the molten metal is the product of its cross-sectional area at the parting line and the pressure of the molten metal due to the height of the sprue. Assume that the sprue has the same height as the cope, namely, 38.1 cm. The pressure of the molten metal is the product of height and density. Assuming a density for the molten metal of 8009.2317 kg/m\(^3\), the pressure at the parting line will be (8009.2317)(0.381) = 3051.52 kg/m\(^2\), or 30 kPa. The buoyancy force is the product of projected area and pressure, or (3051.52)(0.2286)\(^2\) = 500 kg. The net volume of the sand in each flask is

\[
V = (50.8)(50.8)(38.1) - (0.5)\left(\frac{4\pi}{3}\right)(22.86)^3
\]

or \( V = 73,300 \text{ cm}^3 = 0.0733 \text{ m}^3 \). For a sand density of 1601.8463 kg/m\(^3\), the cope weighs 206 kg. Under these circumstances, a clamping force of 1601.8463 – 206 = 855.85 kg is required.
5.62 Would the position of the parting line in Problem 5.61 influence your answer? Explain.

The position of the parting line does have an influence on the answer to Problem 5.54, because (a) the projected area of the molten metal will be different and (b) the weight of the cope will also be different.

5.63 Plot the clamping force in Problem 5.61 as a function of increasing diameter of the casting, from 25.4 cm to 50.8 cm.

Note in this problem that as the diameter of the casting increases, the cross-sectional area of the molten metal increases, hence the buoyancy force also increases. At the same time, the weight of the cope decreases because of the larger space taken up by the molten metal. Using the same approach as in Problem 5.61, the weight of the casting as a function of diameter is given by

\[ F_b = \rho V = 8009.2317 \left( \frac{\pi}{6} d^3 \right) \]

\[ = (4193.6 \text{ kg/m}^3)d^3 \]

The volume of sand in the cope is given by:

\[ V = (50.8)(50.8)(38.1) - (0.5) \left( \frac{\pi}{6} d^3 \right) \]

\[ = 0.0983 \text{ m}^3 - 0.261d^3 \]

Therefore, the weight of the sand is given by:

\[ F_w = \rho_{sand} V \]

\[ = (1601.8463 \text{ kg/m}^3)(0.0983 \text{ m}^3 - 0.261d^3) \]

\[ = 157.5 \text{ kg} - (418 \text{ kg/m}^3)d^3 \]

The required clamping force is given by equilibrium as

\[ F_c = F_b - F_w \]

\[ = (4193.6 \text{ kg/m}^3)d^3 - 157.5 \text{ kg} + (418 \text{ kg/m}^3)d^3 \]

\[ = (4611.6 \text{ kg/m}^3)d^3 - 157.5 \text{ kg} \]

This equation is plotted below. Note that for a small diameter, no clamping force is needed, as the weight of the cope is sufficient to hold the cope and drag together.

5.64 Sketch a graph of specific volume vs. temperature for a metal that shrinks as it cools from the liquid state to room temperature. On the graph, mark the area where shrinkage is compensated for by risers.

The graph is as follows. See also Fig. 5.1b on p. 189.

5.65 A round casting has the same dimensions as in Problem 5.59. Another casting of the same metal is rectangular in cross-section, with a width-to-thickness ratio of 3, and has the same length and cross-sectional area as the round casting. Both pieces are cast under the same conditions. What is the difference in the solidification times of the two castings?

The castings have the same length and cross-sectional area (thus the same volume) and the same casting conditions, hence the same C value. The total surface area of the round casting, with \( l = 500 \text{ mm} \) and \( r = 150 \text{ mm} \), is

\[ A_{\text{round}} = 2\pi l + 2\pi r^2 \]

\[ = 2\pi(150)(500) + 2\pi(150)^2 \]

\[ = 6.13 \times 10^5 \text{ mm}^2 \]
The cross-sectional area of the round casting is 

$$\pi r^2 = \pi (150)^2 = 70,680 \text{ mm}^2.$$ 

The rectangular cross section has sides $x$ and $3x$, so that 

$$70,680 = 3x^2 \quad \rightarrow \quad x = 153 \text{ mm}$$ 

hence the perimeter of the rectangular casting with the same cross-sectional area and axes ratio of 3 is 1228 mm. The total surface area is 

$$A_{\text{rect}} = 2(70,680) + (1228)(500)$$ 

or $A_{\text{rect}} = 7.55 \times 10^5 \text{ mm}^2$. According to Chvorinov’s rule, cooling time for a constant volume is inversely proportional to surface area squared. Therefore, 

$$t_{\text{rect}} = \frac{(6.13 \times 10^5)^2}{7.55 \times 10^5} = 0.66$$

5.66 A 75 mm-thick square plate and a right circular cylinder with a radius of 100 mm and height of 50 mm each have the same volume. If each is to be cast using a cylindrical riser, will each part require the same size riser to ensure proper feeding of the molten metal? Explain.

Recall that it is important for the riser to solidify after the casting has solidified. A casting that solidifies rapidly would most likely require a smaller riser than one which solidifies over a longer period of time. Let’s now calculate the relative solidification times, using Chvorinov’s rule given by Eq. (5.11) on p. 205.

For the cylindrical part, we have 

$$V_{\text{cylinder}} = \pi r^2 h = \pi (0.1 \text{ m})^2 (0.050 \text{ m})$$ 

or $V_{\text{cylinder}} = 0.00157 \text{ m}^3$. The surface area of the cylinder is 

$$A_{\text{cylinder}} = 2\pi r^2 + 2\pi rh = 2\pi (0.1)^2 + 2\pi (0.1)(0.05) = 0.0942 \text{ m}^2$$

Thus, from Eq. (5.11) on p. 205 

$$t_{\text{cylinder}} = C \left( \frac{0.00157}{0.0942} \right)^2 = (2.78 \times 10^{-4}) C$$

For a square plate with sides $L$ and height $h = 0.075 \text{ m}$, and the same volume as the cylinder, we have 

$$V_{\text{plate}} = 0.00157 \text{ m}^3 = L^2 h = L^2 (0.075 \text{ m})$$

Solving for $L$ yields $L = 0.144 \text{ m}$. Therefore, 

$$A_{\text{plate}} = 2L^2 + 4Lh$$

$$= 2(0.144)^2 + 4(0.144)(0.075)$$

or $A_{\text{plate}} = 0.0847 \text{ m}^2$. From Eq. (5.11) on p. 205, 

$$t_{\text{plate}} = C \left( \frac{0.00157}{0.0847} \right)^2 = (3.43 \times 10^{-4}) C$$

Therefore, the cylindrical casting will take longer to solidify and will thus require a larger riser.

5.67 Assume that the top of a round sprue has a diameter of 10.16 cm and is at a height of 30.48 cm from the runner. Based on Eq. (5.7), plot the profile of the sprue diameter as a function of its height. Assume that the sprue has a diameter of 2.54 cm at its bottom.

From Eq. (5.7) on p. 201 and substituting for the area, it can be shown that 

$$\frac{d^2}{dh^2} = \sqrt{\frac{h}{h_1}}$$

Therefore, 

$$d = \sqrt{d_1^2 \sqrt{\frac{h_1}{h}}} \quad \rightarrow \quad d = C h^{-0.25}$$

The difficulty here is that the reference location for height measurements is not known. Often chokes or wells are used to control flow, but this problem will be solved assuming that proper flow is to be attained by considering hydrodynamics in the design of the sprue. The boundary conditions are that at $h = h_o$, $d = 10.16 \text{ cm}$ (where $h_o$ is the height at the top of the sprue from the reference location) and at $h = h_o + 30.48 \text{ cm}$, $d = 2.54 \text{ cm}$. The first boundary condition yields 

$$10.16 = C(h_o)^{-0.25} \quad \text{or} \quad C = 10.16 h_o^{0.25}$$

The secondary boundary condition yields 

$$2.54 = C(h_o + 30.48)^{-0.25}$$

$$= (10.16 h_o^{0.25})(h_o + 30.48)^{-0.25}$$

This equation is solved as $h_o = 0.11938 \text{ cm}$, so that $C = 5.972$. These values are substituted into the expression above to obtain 

$$d = 5.972(h + 0.11938)^{-0.25}$$
5.68 Estimate the clamping force for a diecasting machine in which the casting is rectangular, with projected dimensions of 75 mm x 150 mm. Would your answer depend on whether or not it is a hot-chamber or cold-chamber process? Explain.

The clamping force is needed to compensate for the separating force developed when the metal is injected into the die. When the die is full, and the full pressure is developed, the separating force is \( F = pA \), where \( p \) is the pressure and \( A \) is the projected area of the casting. Note that the answer will depend on whether the operation is hot- or cold-chamber, because pressures are higher in the cold-chamber than in the hot-chamber process.

The projected area is 11,250 mm\(^2\). In the hot-chamber process, an average pressure is taken as 15 MPa (see Section 5.10.3), although the pressure can range up to 35 MPa. If we use an average pressure, the required clamping force is

\[
F_{\text{hot}} = pA = (35)(11,250) = 394 \text{ kN}
\]

For the cold-chamber process and using a mid-range pressure of 45 MPa, the force will be

\[
F_{\text{cold}} = pA = (45)(11,250) = 506 \text{ kN}
\]

5.69 When designing patterns for casting, patternmakers use special rulers that automatically incorporate solid shrinkage allowances into their designs. Therefore, a 30.48 cm patternmaker’s ruler is longer than a foot. How long should a patternmaker’s ruler be for the making of patterns for (1) aluminum castings (2) malleable cast iron and (3) high-manganese steel?

It was stated in Section 5.12.2 on p. 248 that typical shrinkage allowances for metals are 1.042 to 2.084 cm/m, so it is expected that the ruler be around 31.522–32.564 cm long. Specific shrinkage allowances for these metals can be obtained from the technical literature or the internet. For example, from Kalpakjian, *Manufacturing Processes for Engineering Materials*, 3rd ed., p. 280, we obtain the following:

<table>
<thead>
<tr>
<th>Metal</th>
<th>Shrinkage allowance, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>1.3</td>
</tr>
<tr>
<td>Malleable cast Iron</td>
<td>0.89</td>
</tr>
<tr>
<td>High-manganese steel</td>
<td>2.6</td>
</tr>
</tbody>
</table>

From the following formula,

\[
L_{\text{ruler}} = L_0(1 + \text{shrinkage})
\]

We find that for aluminum,

\[
L_{\text{Al}} = (30.48)(1.013) = 30.876 \text{ cm}
\]

For malleable cast iron,

\[
L_{\text{iron}} = (30.48)(1.0089) = 30.75 \text{ cm}
\]

and for high-manganese steel,

\[
L_{\text{steel}} = (30.48)(1.026) = 31.272 \text{ cm}
\]

Note that high-manganese steel and malleable cast iron were selected for this problem because they have extremely high and low shrinkage allowances, respectively. The aluminum ruler falls within the expected range, as do most other metals.

5.70 The blank for the spool shown in the accompanying figure is to be sand cast out of A-319, an aluminum casting alloy. Make a sketch of the wooden pattern for this part. Include all necessary allowances for shrinkage and machining.
The sketch for a typical green-sand casting pattern for the spool is shown below. A cross-sectional view is also provided to clearly indicate shrinkage and machining allowances, as well as the draft angles. The important elements of this pattern are as follows (dimensions in inches):

(a) Two-piece pattern.
(b) Locating pins will be needed in the pattern plate to ensure that these features align properly.
(c) Shrinkage allowance = $\frac{5}{32}$ in./ft.
(d) Machining allowance = $\frac{1}{16}$ in.
(e) Draft $= 3^\circ$.

5.71 Repeat Problem 5.70, but assume that the aluminum spool is to be cast using expendable-pattern casting. Explain the important differences between the two patterns.

A sketch for a typical expandable-pattern casting is shown below. A cross-sectional view is also provided to clearly show the differences between green-sand (from Problem 5.70) and evaporative-casting patterns. There will be some variations in the patterns produced by students depending on which dimensions are assigned a machining allowance. The important elements of this pattern are as follows (dimensions in centimeters):

(a) One-piece pattern, made of polystyrene.
(b) Shrinkage allowance = 1.3 cm/m
(c) Machining allowance = 0.15875 cm
(d) No draft angles are necessary.

5.72 In sand casting, it is important that the cope mold half be held down with sufficient force to keep it from floating when the molten metal is poured in. For the casting shown in the accompanying figure, calculate the minimum amount of weight necessary to keep the cope from floating up as the molten metal is poured in. (Hint: The buoyancy force exerted by the molten metal on the cope is related to the effective height of the metal head above the cope.)
The cope mold half must be sufficiently heavy or be weighted sufficiently to keep it from floating when the molten metal is poured into the mold. The buoyancy force, $F$, on the cope is exerted by the metallostatic pressure (caused by the metal in the cope above the parting line) and can be calculated from the formula

$$ F = pA $$

where $p$ is the pressure at the parting line and $A$ is the projected area of the mold cavity. The pressure is

$$ p = wh = (7.196775 \text{ g/cm}^3)(7.62 \text{ cm}) = 5.5 \text{ kPa} $$

The projected mold-cavity area can be calculated from the dimensions given on cross section AA in the problem, and is found to be $65.355 \text{ cm}^2$. Thus, the force is

$$ F = (5.5)(65.355) = 3.59 \text{ kg} $$

Note that the height of the cylindrical portion is equal to its radius (so that the total height of the riser is equal to its diameter). The volume, $V$, of this riser is

$$ V = \pi r^2 h + \frac{1}{2} \left( \frac{4\pi r^3}{3} \right) = \frac{5\pi r^3}{3} $$

Letting $V$ be unity, we have

$$ r = \left( \frac{3}{5\pi} \right)^{1/3} $$

The surface area of the riser is

$$ A = 2\pi rh + \pi r^2 + \frac{1}{2} (4\pi r^2) = 5\pi r^2 $$

Substituting for $r$, we obtain $A = 5.21$. Therefore, from Eq. (5.11) on p. 205, the solidification time, $t$, for the blind riser will be

$$ t = C \left( \frac{V}{A} \right)^2 = C \left( \frac{1}{5.21} \right)^2 = 0.037C $$

The optimum shape of a riser is spherical to ensure that it cools more slowly than the casting it feeds. Spherically shaped risers, however, are difficult to cast. (1) Sketch the shape of a blind riser that is easy to mold, but also has the smallest possible surface area-to-volume ratio. (2) Compare the solidification time of the riser in part (a) to that of a riser shaped like a right circular cylinder. Assume that the volume of each riser is the same, and that for each the height is equal to the diameter (see Example 5.2).

A sketch of a blind riser that is easy to cast is shown below, consisting of a cylindrical and a hemispherical portion.
From Example 5.2, we know that the solidification time for a cylinder with a height equal to its diameter is 0.033 C. Thus, this blind riser will cool a little slower, but not much so, and is easier to cast.

5.74 The part shown in the accompanying figure is a hemispherical shell used as an acetabular (mushroom shaped) cup in a total hip replacement. Select a casting process for this part and provide a sketch of all patterns or tooling needed if it is to be produced from a cobalt-chrome alloy.

![Dimensions in mm](image)

This is an industrially-relevant problem, as this is the casting used as acetabular cups for total hip replacements. There are several possible answers to this question, depending on the student’s estimates of production rate and equipment costs. In practice, this part is produced through an investment casting operation, where the individual parts with runners are injection molded and then attached to a central sprue. The tooling that would be required include: (1) a mold for the injection molding of wax into the cup shape. (2) Templates for placement of the cup shape onto the sprue, in order to assure proper spacing for even, controlled cooling. (3) Machining fixtures. It should be noted that the wax pattern will be larger than the desired casting, because of shrinkage as well as the incorporation of a shrinkage allowance.

5.75 A cylinder with a height-to-diameter ratio of unity solidifies in four minutes in a sand casting operation. What is the solidification time if the cylinder height is doubled? What is the time if the diameter is doubled?

From Chvorinov’s rule, given by Eq. (5.11) on p. 205, and assuming \( n = 2 \) gives

\[
 t = C \left( \frac{V}{A} \right)^2 \\
 = C \left[ \frac{(\pi d^2 h/4)}{2 + \pi dh} \right]^2 \\
 = C \left( \frac{dh}{2d + 4h} \right) = 4 \text{ min}
\]

Solving for \( C \),

\[
 C = (4 \text{ min}) \left( \frac{2d + 4h}{dh} \right)^2
\]

If the height is doubled, then we can use \( d_2 = d \) and \( h_2 = 2h \) to obtain

\[
 t = C \left( \frac{d_2 h_2}{2d_2 + 4h_2} \right)^2 \\
 = (4 \text{ min}) \left( \frac{2d + 4h}{dh} \right)^2 \left( \frac{d(2h)}{2d + 4(2h)} \right)^2 \\
 = (4 \text{ min}) \left( \frac{4d + 8h}{2d + 8h} \right)^2
\]

If \( d = h \), then

\[
 t = (4 \text{ min}) \left( \frac{12h}{10h} \right)^2 = 5.76 \text{ min}
\]

If the diameter is doubled, so that \( d_3 = 2d \) and \( h_3 = h \), then

\[
 t = C \left( \frac{d_3 h_3}{2d_3 + 4h_3} \right)^2 \\
 = (4 \text{ min}) \left( \frac{2d + 4h}{dh} \right)^2 \left( \frac{(2d)(h)}{2(2d) + 4(h)} \right)^2 \\
 = (4 \text{ min}) \left( \frac{4d + 8h}{4d + 4h} \right)^2
\]

or, for \( d = h \),

\[
 t = (4 \text{ min}) \left( \frac{12h}{8h} \right)^2 = 9 \text{ min}
\]

5.76 Steel piping is to be produced by centrifugal casting. The length is 3.6576 m, the diameter is 0.9144 m, and the thickness is 1.27 cm. Using basic equations from dynamics and statics, determine the rotational speed needed to have the centripetal force be 70 times its weight.
The centripetal force can be obtained from an undergraduate dynamics textbook as

\[ F_c = \frac{mv^2}{r} \]

where \( m \) is the mass, \( v \) is the tangential velocity, and \( r \) is the radius. It is desired to have this force to be 70 times its weight, or

\[ 70 = \frac{F_c}{W} = \frac{mv^2}{mg} = \frac{v^2}{rg} \]

since \( r \) is the mean radius of the casting, or 0.381 m, \( v \) can be solved as

\[ v = \sqrt{(70)rg} = \sqrt{(70)(0.381)(9.81)} \]

or \( v = 16.17 \text{ m/s} \) or 1615.44 cm/s. The rotational speed needed to obtain this velocity is

\[ \omega = \frac{v}{r} = \frac{16.17}{0.381} = 42.4 \text{ rad/s} \]

This is equivalent to 405 rev/min.

5.77 A sprue is 30.48 cm long and has a diameter of 12.7 cm at the top, where the metal is poured. The molten metal level in the pouring basin is taken as 7.62 cm from the top of the sprue for design purposes. If a flow rate of 655.48 cm³/s is to be achieved, what should be the diameter of the bottom of the sprue? Will the sprue aspirate? Explain.

Assuming the flow is frictionless, the velocity of the molten metal at the bottom of the sprue (\( h = 30.48 \text{ cm} \)) is

\[ v = \sqrt{2gh} = \sqrt{2(9.81)(30.48)} \]

or \( v = 2.44 \text{ m/s} = 244 \text{ cm/s} \). For a flow rate of 655.48 cm³/s, the area needs to be

\[ A = \frac{Q}{v} = \frac{655.48 \text{ cm}^3/\text{s}}{244 \text{ cm/s}} = 2.686 \text{ cm}^2 \]

For a circular runner, the diameter would then be 1.854 cm, or roughly 1.905 cm. Compare this to the diameter at the bottom of the sprue based on Eq. (5.7), where \( h_2 = 7.62 \text{ cm} \), \( h_2 = 38.1 \text{ cm} \), and \( A_1 = 126.45 \text{ cm}^2 \). The diameter at the bottom of the sprue is calculated from:

\[ \frac{A_1}{A_2} = \sqrt{\frac{h_2}{h_1}} \]

\[ A_2 = \frac{A_1}{\sqrt{h_2/h_1}} = \frac{126.45}{\sqrt{38.1/7.62}} = 56.55 \text{ cm}^2 \]

Thus, the sprue confines the flow more than is necessary, and it will not aspirate.

5.78 Small amounts of slag often persist after skimming and are introduced into the molten metal flow in casting. Recognizing that the slag is much less dense than the metal, design mold features that will remove small amounts of slag before the metal reaches the mold cavity.

There are several dross-trap designs in use in foundries. (A good discussion of trap design is given in J. Campbell, Castings, 1991, Reed Educational Publishers, pp. 53-55.) A conventional and effective dross trap is illustrated below:

It is designed on the principle that a trap at the end of a runner will take the metal through the runner and keep it away from the gates. The design shown is a wedge-type trap. Metal entering the runner contacts the wedge, and the leading front of the metal wave is chilled and attaches itself to the runner wall, and thus it is kept out of the mold cavity. The wedge must be designed so as to avoid reflected waves that otherwise would recirculate the dross or the slag.

The following design is a swirl trap:
This is based on the principle that the dross or slag is less dense than the metal. The metal enters the trap off of the center, inducing a swirl in the molten metal as the trap fills with molten metal. Because it is far less dense than the metal, the dross or slag remains in the center of the swirl trap. Since the metal is tapped from the outside periphery, dross or slag is excluded from the casting.

5.79 Pure aluminum is being poured into a sand mold. The metal level in the pouring basin is 25.4 cm above the metal level in the mold, and the runner is circular with a 1.016 cm diameter. What is the velocity and rate of the flow of the metal into the mold? Is the flow turbulent or laminar?

Equation (5.5) on p. 200 gives the metal flow. Assuming the pressure does not change appreciably in the channel and that there is no friction in the sprue, the flow is

\[ h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} \]

Where the subscript 1 indicates the top of the sprue and 2 the bottom. If we assume that the velocity at the top of the sprue is very low (as would occur with the normal case of a pouring basin on top of the sprue with a large cross-sectional area), then \( v_1 = 0 \). The velocity at the bottom of the sprue is

\[ v_2^2 = 2g(h_1 - h_2) \]

or

\[ v_2 = \sqrt{2g(h_1 - h_2)} \]

or \( v_2 = 223.2 \text{ cm/s} \). If the opening is 1.016 cm in diameter, the area is

\[ A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (1.016)^2 = 0.81 \text{ cm}^2 \]

Therefore, the flow rate is

\[ Q = v_2 A = (223.2)(0.81) = 180.955 \text{ cm}^3/\text{s} \]

Pure aluminium has a density of 2700 kg/m\(^3\) (see Table 3.3) and a viscosity of around 0.0015 Ns/m\(^2\) around 973 K. The Reynolds number, from Eq. (5.10) on p. 202, is then (using \( v = 2.23 \text{ m/s} \) and \( D = 0.01016 \text{ m} \)),

\[ \text{Re} = \frac{vD \rho}{\eta} \]

\[ = \frac{(2.23 \text{ m/s})(0.01016 \text{ m})(2700 \text{ kg/m}^3)}{0.0015 \text{ Ns/m}^2} \]

or \( \text{Re} = 40,782 \). As discussed in Section 5.4.1 starting on p. 199, this situation would represent turbulence, and the velocity and/or diameter should be decreased to bring Re below 20,000 or so.

5.80 For the sprue described in Problem 5.79, what runner diameter is needed to ensure a Reynolds number of 2000? How long will a 0.508 m\(^3\) casting take to fill with such a runner?

Using the data given in Problem 5.79, a Reynolds number of 2000 can be achieved by reducing the channel diameter, so that

\[ \text{Re} = 2000 = \frac{vD \rho}{\eta} = \frac{(2.23)(2700)}{0.0015} \frac{\text{m}}{D} \]

or \( D = 0.000498 \text{ m} \).

For this diameter, the initial flow rate would be

\[ Q = v_2 A = \frac{\pi v_2}{4} D^2 = \frac{\pi}{4} (223.2)(0.000498)^2 \]

\[ = 0.435 \text{ cm}^3/\text{s} \]

This means that a 327.74 cm\(^3\) casting would take 753 s (about 12 min) to fill and only if the initial flow rate could be maintained, which is generally not the case. Such a long filling time is not acceptable, since it is likely that metal will solidify in runners and thus not fill the mold completely. Also, with such a small runner, additional mechanisms need to be considered. For example, surface tension and friction would severely reduce the velocity in the Reynolds number calculation above.

This is generally the case with castings; to design a sprue and runner system that maintains laminar flow in the fluid would result in excessively long fill times.

5.81 How long would it take for the sprue in Problem 5.79 to feed a casting with a square cross section of 15.24 cm per side and a height of 10.16 cm? Assume the sprue is frictionless.
Note that the volume of the casting is 2360 cm³, with a constant cross-sectional area of 232.26 cm². The velocity will change as the mold fills, because the pouring basin height above the molten metal will decrease. The velocity will vary according to:

\[ v = c\sqrt{2gh} = \sqrt{2gh} \]

The flow rate is given by

\[ Q = vA = \frac{\pi d^2}{4} = \frac{\pi d^2\sqrt{2gh}}{4} \]

The mold cavity fills at a rate of \( Q/(232.26\text{ cm}^2) \), or

\[ \frac{dh}{dt} = \frac{Q}{A} = -\frac{\pi d^2\sqrt{2gh}}{4A} \]

where the minus sign has been added so that \( h \) refers to the height difference between the metal level in the mold and the runner, which decreases with respect to time. Separating the variables,

\[ \frac{dh}{\sqrt{h}} = -\frac{\pi d^2\sqrt{2g}}{4A} dt \]

Integrating,

\[ \left(2\sqrt{h}\right)_{15.24\text{ cm}}^{25.4\text{ cm}} = \frac{\pi d^2\sqrt{2g}}{4A} (t)_0 \]

From this equation and using \( d = 1.016 \text{ cm} \) and \( A = 232.26 \text{ cm}^2 \), \( t \) is found to be 14.7 s. As a comparison, using the flow rate calculated in Problem 5.79, the mold would require approximately 13 s to fill.

5.82 A rectangular mold with dimensions 100 mm × 200 mm × 400 mm is filled with aluminum with no superheat. Determine the final dimensions of the part as it cools to room temperature. Repeat the analysis for gray cast iron.

Note that the initial volume of the box is \((0.100)(0.200)(0.400)=0.008 \text{ m}^3\). From Table 3.3 on p. 206, the volumetric contraction for aluminum is 6.6%. Therefore, the box volume will be

\[ V = (1 - 0.066)(0.008 \text{ m}^3) = 0.007472 \text{ m}^3 \]

Assuming the box has the same aspect ratio as the mold (1:2:4) and that warpage can be ignored, we can calculate the dimensions of the box after solidification as 97.7 mm × 195.5 mm × 391 mm. From Table 3.3 on p. 106, the melting point of aluminum is 660°C, with a coefficient of thermal expansion of 23.6 µm/m°C. Thus, the total strain in cooling from 660°C to room temperature (25°C) is

\[ \epsilon = \alpha \Delta t = (23.6 \mu\text{m}/\text{m}^\circ\text{C})(660^\circ\text{C} - 25^\circ\text{C}) \]

or \( \epsilon = 0.0150 \). The final box dimensions are therefore 96.2 mm × 192.5 mm × 385 mm.

For gray cast iron, the metal expands upon solidification. Assuming the mold will allow for expansion, the volume after solidification is given by

\[ V = (1.025)(0.008 \text{ m}^3) = 0.0082 \text{ m}^3 \]

If the box has the same aspect ratio as the initial mold cavity, the dimensions after solidification will be 100.8 × 201.7 × 403.3 mm. Using the data for iron in Table 3.3, the melting point is taken as 1537°C and the coefficient of thermal expansion as 11.5 µm/m°C. Therefore,

\[ \epsilon = \alpha \Delta t = (11.5 \mu\text{m}/\text{m}^\circ\text{C})(1537^\circ\text{C} - 25^\circ\text{C}) \]

or \( \epsilon = 0.0174 \). Hence, the final dimensions are 99.0 × 198.1 × 396 mm. Note that even though the cast iron had to cool off from a higher initial temperature, the box of cast iron is much closer to the mold dimensions than the aluminum.

5.83 The constant \( C \) in Chvorinov’s rule is given as 3 s/mm² and is used to produce a cylindrical casting with a diameter of 75 mm and a height of 125 mm. Estimate the time for the casting to fully solidify. The mold can be broken safely when the solidified shell is at least 20 mm. Assuming the cylinder cools evenly, how much time must transpire after pouring the molten metal before the mold can be broken?

Note that for the cylinder

\[ A = 2\left(\frac{\pi d^2}{4}\right) + \pi dh \]

\[ = 2\left[ \frac{\pi}{4}(75)^2 \right] + \pi(75)(125) \]

\[ = 38,290 \text{ mm}^2 \]

\[ V = \frac{\pi}{4}d^2h = \frac{\pi}{4}(75)^2(125) = 5.522 \times 10^5 \text{ mm}^3 \]
A jeweler wishes to produce twenty gold rings in one investment-casting operation. The wax parts are attached to a wax central sprue of a 12.7 cm diameter. The rings are located in four rows, each 12.7 cm from the other on the sprue. The rings require a 0.3175 cm diameter and 12.7 cm-long runner to the sprue. Estimate the weight of gold needed to completely fill the rings, runners, and sprues. The specific gravity of gold is 19.3.

The particular answer will depend on the geometry selected for a typical ring. Let's approximate a typical ring as a tube with dimensions of 2.54 cm outer diameter, 1.5875 cm inner diameter, and 0.9525 cm width. The volume of each ring is then 2.95 cm³, and a total volume for 20 rings of 59 cm³. There are twenty runners to the sprue, so this volume component is

\[ V = 20 \left( \frac{\pi}{4} d^2 \right) L \]

or \( V = 2 \text{ cm}^3 \). The central sprue has a length of 3.81 cm, so that its volume is

\[ V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} (1.27 \text{ cm})^2 (3.81 \text{ cm}) = 4.83 \text{ cm}^3 \]

The total volume is then 65.55 cm³, not including the metal in the pouring basin, if any. The specific gravity of gold is 19.3, thus its density is 19.3(1000 kg/m³) = 19,300 kg/m³. Therefore, the jeweler needs 1.265 kg of gold.

Assume that you are asked to give a quiz to students on the contents of this chapter. Prepare three quantitative problems and three qualitative questions, and supply the answers.

By the student. This is a challenging, open-ended question that requires considerable focus and understanding on the part of the students, and has been found to be a very valuable homework problem.

Design

5.87 Design test methods to determine the fluidity of metals in casting (see Section 5.4.2 starting on p. 203). Make appropriate sketches and explain the important features of each design.
By the student. The designs should allow some method of examining the ability of a metal to fill the mold. One example, taken from Kalpakjian and Schmid, *Manufacturing Engineering and Technology*, 5th ed, Prentice-Hall, 2001, is shown below.

5.88 The accompanying figures indicate various defects and discontinuities in cast products. Review each one and offer design solutions to avoid them.

By the student. Some examples are for (a) fracture is at stress raiser, a better design would utilize a more gradual filet radius; (b) fracture at the gate indicates this runner section is too narrow and thus it solidified first, hence the gate should be larger.

5.89 Utilizing the equipment and materials available in a typical kitchen, design an experiment to reproduce results similar to those shown in Fig. 5.12.

A simple experiment can be performed with melted chocolate and a coffee cup. If a parting agent is sprayed into the cup, and molten chocolate is poured, after a short while the still molten center portion can be poured from the cup, leaving a solidified shell. This effect can be made more pronounced by using chilling the cups first.

5.90 Design a test method to measure the permeability of sand for sand casting.

Permeability suggests that there is a potential for material to penetrate somewhat into the porous mold material. The penetration can be measured through experimental setups, such as using a standard-sized slug or shape of sand, applying a known pressure to one side, and then measuring the flow rate through the sand.

5.91 Describe the procedures that would be involved in making a bronze statue. Which casting process or processes would be suitable? Why?

The answer depends on the size of the statue. A small statue (say 100 mm tall) can be die cast if the quantities desired are large enough, or it can be sand cast for fewer quantities. The very large statues such as those found in public parks, which typically are on the order of 1 to 3 m tall, are produced by first manufacturing or sculpting a blank from wax and then using the investment-casting process. Another option for a large casting is to carefully prepare a ceramic mold.

5.92 Porosity developed in the boss of a casting is illustrated in the accompanying figure. Show that by simply repositioning the parting line of this casting, this problem can be eliminated.
Note in the figure that the boss is at some distance from the blind riser. Consequently, the boss can develop porosity (not shown in the figure, but to be added by the instructor) because of a lack of supply of molten metal from the riser. The sketch below shows a repositioned parting line that would eliminate porosity in the boss. Note in the illustration below that the boss can now be supplied with molten metal as it begins to solidify and shrink.

5.93 For the wheel illustrated in the accompanying figure, show how (a) riser placement, (b) core placement, (c) padding, and (d) chills may be used to help feed molten metal and eliminate porosity in the isolated hob boss.

Four different methods are shown below.

5.94 In the figure below, the original casting design shown in (a) was changed to the design shown in (b). The casting is round, with a vertical axis of symmetry. As a functional part, what advantages do you think the new design has over the old one?

By the student. There are several advantages, including that the part thickness is more uniform, so that large shrinkage porosity is less likely, and the ribs will control warping due to thermal stresses as well as increasing joint stiffness.

5.95 An incorrect and a correct design for casting are shown, respectively, in the accompanying figure. Review the changes made and comment on their advantages.

By the student. The main advantage of the new design is that it can be easily cast without the need for an external core. The original part would require two such cores, because the geometry is such that it cannot be obtained in a sand mold without cores.
5.96 Three sets of designs for die casting are shown in the accompanying figure. Note the changes made to original die design (number 1 in each case) and comment on the reasons.

By the student. There are many observations, usually with the intent of minimizing changes in section thickness, eliminating inclined surfaces to simplify mold construction, and to orient flanges so that they can be easily cast.

5.97 It is sometimes desirable to cool metals more slowly than they would be if the molds were maintained at room temperature. List and explain the methods you would use to slow down the cooling process.

There can be several approaches to this problem, including:

- Heated molds will maintain temperatures higher than room temperature, but will still allow successful casting if the mold temperature is below the melting temperature of the metal.
- The mold can be placed in a container; heat from the molten metal will then warm the local environment above room temperature.
- The mold can be insulated to a greater extent, so that its steady-state temperature is higher (permanent-mold processes).
- The mold can be heated to a higher temperature.
- An exothermic jacket can be placed around the molten metal.
- Radiation heat sources can be used to slow the rate of heat loss by conduction.

5.98 Design an experiment to measure the constants $C$ and $n$ in the Chvorinov’s Rule [Eq. (5.11)]. The following are some tests that could be considered:

- The most straightforward tests involve producing a number of molds with a family of parts (such as spheres, cubes or cylinders with a fixed length-to-diameter ratio), pouring them, and then breaking the mold periodically to observe if the metal has solidified. This inevitably results in spilled molten metal and may therefore a difficult test procedure to use.
- Students should consider designing molds that are enclosed but have a solidification front that terminates at an open riser; they can then monitor the solidification times can then be monitored, and then determine fit Eq. (5.11) on p. 205 to their data.
- An alternative to solidifying the metal is to melt it within a mold specially designed for such an experiment.

5.99 The part in the accompanying figure is to be cast of 10% Sn bronze at the rate of 100 parts per month. To find an appropriate casting process, consider all the processes in this chapter, then reject those that are (a) technically inadmissible, (b) technically feasible but too expensive for the purpose, and (c) identify the
most economical process. Write a rationale using common-sense assumptions about product cost.

The answers could be somewhat subjective, because the particular economics are affected by company capabilities and practices. The following summary is reasonable suggestion:

<table>
<thead>
<tr>
<th>Process</th>
<th>Note</th>
<th>Cost rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand casting</td>
<td></td>
<td>This is probably best.</td>
</tr>
<tr>
<td>Shell-mold casting</td>
<td>(a)</td>
<td>Need tooling to make blanks. Too low of production rate to justify.</td>
</tr>
<tr>
<td>Lost Foam</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plaster mold</td>
<td>(a)</td>
<td></td>
</tr>
<tr>
<td>Ceramic mold</td>
<td>(b)</td>
<td>Need to make blanks. Too low of production rate to justify, unless rapid tooling is used.</td>
</tr>
<tr>
<td>Lost Wax</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacuum casting</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>Pressure casting</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>Die casting</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>Centrifugal casting</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>CZ Process</td>
<td>(b)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (a) technically inadmissible; (b) too expensive.