5. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = +4.0 \times 10^{-6}$C.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_\omega = \left(10\times10^{-6}\ \mathrm{C}\right) - \left(+4.0\times10^{-6}\ \mathrm{C}\right) = +6.0\times10^{-6}\ \mathrm{C}.$$

17. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with a proton of charge $q = +1.6\times10^{-19}$ C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \ \text{N}\cdot\text{m}^2/\text{C}.$$

20. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density $\sigma = 2.31\times 10^{-22}$ C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0 = (2.31 \times 10^{-22}\ \text{C/m}^2)/(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 2.61\times10^{-11}$ N/C, pointing in the upward direction, or $\vec{E} = (2.61\times10^{-11}\ \text{N/C})\hat{\text{j}}$;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.61\times10^{-11}\ \text{N/C})\hat{\text{j}}$.

29. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\Phi = \int \vec{E} \cdot d\vec{A} = 4\pi r^2 E,$ where r is the radius of the Gaussian surface.

For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(a) For $r = 0$, the above equation implies $E = 0$.

(b) For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-15}\,\text{C})}{2(2.00\times10^{-2}\,\text{m})^2} = 2.25\times10^{-2}\ \text{N/C}.$$

(c) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-15}\,\text{C})}{(2.00\times10^{-2}\,\text{m})^2} = 0.0450\ \text{N/C}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \Rightarrow E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(d) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-15}\,\text{C})}{(1.50\times2.00\times10^{-2}\,\text{m})^2} = 0.0200\ \text{N/C} = 20.0\ \text{mN/C}.$$

(e) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$.

(f) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0.$ Thus, $E = 0$ at $r = 3.50a$.

(g) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\Phi = \int \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -2.00$ fC.

(h) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0.$$

36. We reason that point P (the point on the x axis where the net electric field is zero) cannot be between the lines of charge (since their charges have opposite sign). We reason further that P is not to the left of "line 1" since its magnitude of charge (per unit length) exceeds that of "line 2"; thus, we look in the region to the right of "line 2" for P. Using Eq. 23-12, we have

$$E_{\text{net}} = E_1 + E_2 = \frac{2\lambda_1}{4\pi\varepsilon_0 (x + L/2)} + \frac{2\lambda_2}{4\pi\varepsilon_0 (x - L/2)} \ .$$

Setting this equal to zero and solving for x we find

$$x = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)\frac{L}{2} = \left(\frac{6.0\mu\text{C/m} - (-2.0\mu\text{C/m})}{6.0\mu\text{C/m} + (-2.0\mu\text{C/m})}\right)\frac{10.0 \text{ cm}}{2} = 10.0 \text{ cm} \ .$$

54. Equation 23-6 (Gauss' law) gives $\varepsilon_o \Phi = q_{enc}$.

(a) Thus, the value $\Phi = 4.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$ for small r leads to

$$q_{\text{central}} = \varepsilon_0 \Phi = (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}) = 3.54 \times 10^{-6} \text{C} \approx 3.5\ \mu\text{C}.$$

(b) The next value that the flux Φ takes is $\Phi = -8.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$, which implies that $q_{\text{enc}} = -7.08 \times 10^{-6}\text{C}$. But we have already accounted for some of that charge in part (a), so the result for part (b) is

$$q_A = q_{\text{enc}} - q_{\text{central}} = = -7.08 \times 10^{-6}\text{C} - 3.54 \times 10^{-6}\text{C} = -10.62 \times 10^{-6}\text{C} \approx -11\ \mu\text{C}.$$

(c) Finally, the large r value for the flux Φ is $\Phi = 12.0 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$, which implies that $q_{\text{total enc}} = 10.62 \times 10^{-6}\text{C}$. Considering what we have already found, then the result is

$$q_B = q_{\text{total enc}} - q_A - q_{\text{central}} = 10.62\ \mu\text{C} - (-10.62\ \mu\text{C}) - 3.54\ \mu\text{C} = +17.7\ \mu\text{C} \approx +18\ \mu\text{C}.$$