Chapter 29

1. (a) Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and sin θ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x = -L/2$ to $x = L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{\left(x^2+R^2\right)^{1/2}}\Bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}$$

$$= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(0.0582\ \text{A}\right)}{2\pi\left(0.240\ \text{m}\right)}\frac{0.180\text{m}}{\sqrt{(0.180\text{m})^2+4(0.240\ \text{m})^2}} = 1.70\times10^{-8}\ \text{T}.$$

(b) By right-hand-rule, the magnetic field at P points out of the page.

(c) As can be seen from the expression above, increasing R decreases B.

60. We note that when there is no y-component of magnetic field from wire 1 (which, by the right-hand rule, relates to when wire 1 is at 90º = $\pi/2$ rad), the total y-component of magnetic field is zero (see Fig. 29-58(c)). This means wire #2 is either at $+\pi/2$ rad or $-\pi/2$ rad.

(a) We now make the assumption that wire #2 must be at –π/2 rad (–90º, the bottom of the cylinder) since it would pose an obstacle for the motion of wire #1 (which is needed to make these graphs) if it were anywhere in the top semicircle.

(b) Looking at the θ_1 = 90º datum in Fig. 29-58(b)), where there is a *maximum* in $B_{\text{net}\,x}$ (equal to +6 μT), we are led to conclude that $B_{1x} = 6.0\ \mu\text{T} - 2.0\ \mu\text{T} = 4.0\ \mu\text{T}$ in that situation. Using Eq. 29-4, we obtain

$$i_1 = \frac{2\pi R B_{1x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(4.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 4.0\text{ A}\,.$$

(c) The fact that Fig. 29-58(b) increases as θ_1 progresses from 0 to 90º implies that wire 1's current is *out of the page*, and this is consistent with the cancellation of $B_{\text{net}\,y}$ at $\theta_1 = 90°$, noted earlier (with regard to Fig. 29-58(c)).

(d) Referring now to Fig. 29-58(b) we note that there is no x-component of magnetic field from wire 1 when θ_1 = 0, so that plot tells us that B_{2x} = +2.0 μT. Using Eq. 29-4, we find the magnitudes of the current to be

$$i_2 = \frac{2\pi R B_{2x}}{\mu_0} = \frac{2\pi(0.200\text{ m})(2.0\times10^{-6}\text{ T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 2.0\text{ A}\,.$$

(e) We can conclude (by the right-hand rule) that wire 2's current is *into the page*.

(b) The direction is out of the page.

8. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in segments *AH* and *JD* do not contribute to the field at point *C*. Using Eq. 29-9 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc HJ contributes $\mu_0 i/4R_1$ (into the page) to the field at *C*. Also, arc DA contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at *C* is

$$B=\frac{\mu_0 i}{4}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.281\text{A})}{4}\left(\frac{1}{0.0315\text{m}}-\frac{1}{0.0780\text{m}}\right)=1.67\times10^{-6}\,\text{T}.$$

(b) The direction of the field is into the page.

9. (a) The currents must be opposite or antiparallel, so that the resulting fields are in the same direction in the region between the wires. If the currents are parallel, then the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires.

(b) At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i=\frac{\pi r B}{\mu_0}=\frac{\pi(0.040\text{ m})\left(300\times10^{-6}\,\text{T}\right)}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}}=30\,\text{A}.$$

10. (a) Recalling the *straight sections* discussion in Sample Problem — "Magnetic field at the center of a circular arc of current," we see that the current in the straight segments collinear with *C* do not contribute to the field at that point.

Equation 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at *C*. Thus, the magnitude of the magnetic field is

$$B=\frac{\mu_0 i}{4R}=\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.0348\text{A})}{4(0.0926\text{m})}=1.18\times10^{-7}\,\text{T}.$$

(b) The right-hand rule shows that this field is into the page.

11. (a) $B_{P_1}=\mu_0 i_1/2\pi r_1$ where i_1 = 6.5 A and $r_1 = d_1 + d_2$ = 0.75 cm + 1.5 cm = 2.25 cm, and $B_{P_2}=\mu_0 i_2/2\pi r_2$ where $r_2 = d_2$ = 1.5 cm. From $B_{P1} = B_{P2}$ we get

$$i_2=i_1\left(\frac{r_2}{r_1}\right)=(6.5\,\text{A})\left(\frac{1.5\text{ cm}}{2.25\text{ cm}}\right)=4.3\,\text{A}.$$

$$B_f^2 = B_z^2 + B_y^2 = \left(\frac{\mu_0 i\phi}{4\pi R}\right)^2 + \left(\frac{\mu_0 i\phi}{4\pi r}\right)^2 .$$

If we square B_i and divide by B_f^2, we obtain

$$\left(\frac{B_i}{B_f}\right)^2 = \frac{[(1/R)+(1/r)]^2}{(1/R)^2+(1/r)^2} .$$

From the graph (see Fig. 29-59(c), note the maximum and minimum values) we estimate $B_i/B_f = 12/10 = 1.2$, and this allows us to solve for r in terms of R:

$$r = R\frac{1 \pm 1.2\sqrt{2-1.2^2}}{1.2^2-1} = 2.3 \text{ cm} \quad \text{or} \quad 43.1 \text{ cm}.$$

Since we require $r < R$, then the acceptable answer is $r = 2.3$ cm.

33. Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w}.$$

Thus,

$$B_P = \int dB_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w}\frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.61\times10^{-6}\ \text{A})}{2\pi(0.0491\ \text{m})}\ln\left(1+\frac{0.0491}{0.0216}\right)$$
$$= 2.23\times10^{-11}\text{T}.$$

and $\vec{B}_P$ points upward. In unit-vector notation, $\vec{B}_P = (2.23\times10^{-11}\text{T})\hat{\text{j}}$

Note: In the limit where $d \gg w$, using

$$\ln(1+x) = x - x^2/2 + \cdots,$$

the magnetic field becomes

$$B_P = \frac{\mu_0 i}{2\pi w}\ln\left(1+\frac{w}{d}\right) \approx \frac{\mu_0 i}{2\pi w}\cdot\frac{w}{d} = \frac{\mu_0 i}{2\pi d}$$

which is the same as that due to a thin wire.

34. By the right-hand rule (which is "built-into" Eq. 29-3) the field caused by wire 1's current, evaluated at the coordinate origin, is along the $+y$ axis. Its magnitude B_1 is given by Eq. 29-4. The field caused by wire 2's current will generally have both an x and a y component, which are related to its magnitude B_2 (given by Eq. 29-4), and sines and

$$F_{\perp \text{ sides}} = \int_a^{a+b} \frac{i_2 \mu_0 i_1}{2\pi y}\, dy.$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a+d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a+b)}$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(30.0\text{A})(20.0\text{A})(8.00\text{cm})\left(300\times10^{-2}\,\text{m}\right)}{2\pi(1.00\text{cm}+8.00\text{cm})} = 3.20\times10^{-3}\,\text{N},$$

and $\vec{F}$ points toward the wire, or $+\hat{\text{j}}$. That is, $\vec{F} = (3.20\times10^{-3}\,\text{N})\hat{\text{j}}$ in unit-vector notation.

42. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\oint_c \vec{B}\cdot d\vec{s} = \mu_0 i = \mu_0 jA = \left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(15\,\text{A/m}^2\right)(6)(0.20\text{m})^2 = 4.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

43. We use Eq. 29-20 $B = \mu_0 i r / 2\pi a^2$ for the B-field inside the wire $(r < a)$ and Eq. 29-17 $B = \mu_0 i / 2\pi r$ for that outside the wire $(r > a)$.

(a) At $r = 0$, $B = 0$.

(b) At $r = 0.0100\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0100\text{m})}{2\pi(0.0200\text{m})^2} = 8.50\times10^{-4}\,\text{T}.$

(c) At $r = a = 0.0200\text{m}$, $B = \dfrac{\mu_0 i r}{2\pi a^2} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})(0.0200\text{m})}{2\pi(0.0200\text{m})^2} = 1.70\times10^{-3}\,\text{T}.$

(d) At $r = 0.0400\text{m}$, $B = \dfrac{\mu_0 i}{2\pi r} = \dfrac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(170\text{A})}{2\pi(0.0400\text{m})} = 8.50\times10^{-4}\,\text{T}.$

44. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop.

(a) For path 1, the result is

$$\oint_1 \vec{B}\cdot d\vec{s} = \mu_0(-5.0\text{A}+3.0\text{A}) = (4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(-2.0\text{A}) = -2.5\times10^{-6}\,\text{T}\cdot\text{m}.$$

(b) For path 2, we find

$$B_y = \frac{\mu_0 i_1 R^2}{2\pi(R^2+z_1^2)^{3/2}} - \frac{\mu_0 i_2 R^2}{2\pi(R^2+z_2^2)^{3/2}},$$

where $z_1{}^2 = L^2$ (see Fig. 29-73(a)) and $z_2{}^2 = y^2$ (because the central axis here is denoted y instead of z). The fact that there is a minus sign between the two terms, above, is due to the observation that the datum in Fig. 29-73(b) corresponding to $B_y = 0$ would be impossible without it (physically, this means that one of the currents is clockwise and the other is counterclockwise).

(a) As $y \to \infty$, only the first term contributes and (with $B_y = 7.2 \times 10^{-6}$ T given in this case) we can solve for i_1. We obtain $i_1 = (45/16\pi)$ A ≈ 0.90 A.

(b) With loop 2 at $y = 0.06$ m (see Fig. 29-73(b)) we are able to determine i_2 from

$$\frac{\mu_0 i_1 R^2}{2(R^2+L^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2+y^2)^{3/2}}.$$

We obtain $i_2 = (117\sqrt{13}/50\pi)$ A ≈ 2.7 A.

61. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(15\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 7.9\times10^{-5}\,\mathrm{T}.$$

(b) The torque has magnitude equal to

$$\begin{aligned}\tau &= |\vec{\mu}_2\times\vec{B}_1| = \mu_2 B_1\sin 90° = N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1\\ &= \pi(50)(1.3\,\mathrm{A})(0.82\times10^{-2}\,\mathrm{m})^2(7.9\times10^{-5}\,\mathrm{T})\\ &= 1.1\times10^{-6}\,\mathrm{N\cdot m}.\end{aligned}$$

62. (a) To find the magnitude of the field, we use Eq. 29-9 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$\begin{aligned}B &= \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.0562\,\mathrm{A})}{4}\left(\frac{1}{0.0572\,\mathrm{m}}+\frac{1}{0.0936\,\mathrm{m}}\right)\\ &= 4.97\times10^{-7}\,\mathrm{T}.\end{aligned}$$

(b) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 29-6(c)).

(c) The enclosed area is $A = (\pi a^2 + \pi b^2)/2$, which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}(a^2+b^2) = \frac{\pi(0.0562\text{A})}{2}[(0.0572\text{m})^2+(0.0936\text{m})^2] = 1.06\times10^{-3}\,\text{A}\cdot\text{m}^2.$$

(d) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper.

63. By imagining that each of the segments *bg* and *cf* (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(a) The dipole moment of path *abcdefgha* is

$$\vec{\mu} = \vec{\mu}_{bcfgb} + \vec{\mu}_{abgha} + \vec{\mu}_{cdefc} = \left(ia^2\right)\left(\hat{\text{j}} - \hat{\text{i}} + \hat{\text{i}}\right) = ia^2\hat{\text{j}}$$
$$= \left(6.0\,\text{A}\right)\left(0.10\,\text{m}\right)^2\hat{\text{j}} = (6.0\times10^{-2}\,\text{A}\cdot\text{m}^2)\,\hat{\text{j}}\,.$$

(b) Since both points are far from the cube we can use the dipole approximation. For $(x, y, z) = (0, 5.0\text{ m}, 0)$,

$$\vec{B}(0,\,5.0\text{ m},\,0) \approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} = \frac{(1.26\times10^{-6}\text{ T}\cdot\text{m/A})(6.0\times10^{-2}\text{m}^2\cdot\text{A})\,\hat{\text{j}}}{2\pi(5.0\text{ m})^3} = (9.6\times10^{-11}\text{ T})\,\hat{\text{j}}\,.$$

64. (a) The radial segments do not contribute to $\vec{B}_P$, and the arc segments contribute according to Eq. 29-9 (with angle in radians). If $\hat{\text{k}}$ designates the direction "out of the page" then

$$\vec{B}_P = \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(4.00\text{ m})}\hat{\text{k}} - \frac{\mu_0 i(7\pi/4\text{ rad})}{4\pi(2.00\text{ m})}\hat{\text{k}}$$

where $i = 0.200$ A. This yields $\vec{B} = -2.75\times10^{-8}\,\hat{\text{k}}$ T, or $|\vec{B}| = 2.75\times10^{-8}$ T.

(b) The direction is $-\hat{\text{k}}$, or into the page.

65. Using Eq. 29-20,

$$|\vec{B}| = \left(\frac{\mu_0 i}{2\pi R^2}\right)r\,,$$

we find that $r = 0.00128$ m gives the desired field value.

66. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B = 0.050$ m wire B. Using Eq. 29-4, we have