4 (a) The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n\pi R^2\right)}{i_s} = \mu_0 \pi R^2 nN \ .$$

(b) As long as the magnetic field of the solenoid is entirely contained within the cross section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

7. THINK **Increasing the separation between the two loops changes the flux through the smaller loop and, therefore, induces a current in the smaller loop.**

EXPRESS The magnetic flux through a surface is given by $\Phi_B = \int \vec{B} \cdot d\vec{A}$, where $\vec{B}$ is the magnetic field and $d\vec{A}$ is a vector of magnitude dA that is normal to a differential area dA. In the case where $\vec{B}$ is uniform and perpendicular to the plane of the loop, $\Phi_B = BA$.

In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Equation 29-27, with $z = x$ (taken to be much greater than R), gives $\vec{B} = \frac{\mu_0 i R^2}{2x^3}\hat{\text{i}}$, where the $+x$ direction is upward in Fig. 30-47. The area of the smaller loop is $A = \pi r^2$.

ANALYZE (a) The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area of the smaller loop:

$$\Phi_B = BA = \frac{\pi \mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi \mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi \mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi \mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases. The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

LEARN The situation in this problem is like that shown in Fig. 30-5(d). The induced magnetic field is in the same direction as the initial magnetic field.

24. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt}=\frac{d\left(\frac{1}{2}Li^2\right)}{dt}=Li\frac{di}{dt}=L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0\text{ H}/12\ \Omega = 0.17\text{ s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.1 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.6 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.7 \times 10^2\text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

40. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi}\ln\left(\frac{r+b/2}{r-b/2}\right).$$

When $r = 1.5b$, we have

$$|\Phi_B| = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(6.9\,\mathrm{A})(0.022\,\mathrm{m})}{2\pi}\ln(2.0) = 2.1\times10^{-8}\,\mathrm{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$\begin{aligned} i_{\text{loop}} &= \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 i a}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+b/2}{r-b/2}\right)\right| = \frac{\mu_0 i a b v}{2\pi R[r^2-(b/2)^2]} \\ &= \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(6.9\,\mathrm{A})(0.022\,\mathrm{m})(0.0080\,\mathrm{m})(3.2\times10^{-3}\,\mathrm{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\,\mathrm{m})^2]} \\ &= 1.4\times10^{-5}\,\mathrm{A}. \end{aligned}$$

$$P_{\text{thermal}}\Delta t = \frac{\varepsilon^2 \Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2B^2}{R\Delta t}$$
$$= \frac{(2.00\times10^{-4}\,\text{m}^2)^2\,(17.0\times10^{-6}\,\text{T})^2}{(5.21\times10^{-6}\,\Omega)\,(2.96\times10^{-3}\,\text{s})}$$
$$= 7.50\times10^{-10}\,\text{J}.$$

33. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux is

$$d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r}\,xdr.$$

By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\varepsilon = \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)$$
$$= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(100\,\text{A}\right)\left(5.00\,\text{m/s}\right)}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) = 2.40\times10^{-4}\,\text{V}.$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon/R = \left(2.40\times10^{-4}\,\text{V}\right)/\left(0.400\,\Omega\right) = 6.00\times10^{-4}\,\text{A}.$$

Since the flux is increasing, the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = \left(6.00\times10^{-4}\,\text{A}\right)^2\left(0.400\,\Omega\right) = 1.44\times10^{-7}\,\text{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is

$$dF_B = i_\ell B\,dr = \left(\mu_0 i_\ell i / 2\pi r\right) dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned} F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right) \\ &= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(6.00\times10^{-4}\,\text{A}\right)\left(100\,\text{A}\right)}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) \\ &= 2.87\times10^{-8}\ \text{N}. \end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of 2.87×10^{-8} N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\ \text{N})(5.00\ \text{m/s}) = 1.44 \times 10^{-7}\ \text{W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

34. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R}\ ,$$

which yields $v_t = mgR/B^2L^2$.

35. (a) Equation 30-8 leads to

$$\varepsilon = BLv = (1.2\,\text{T})(0.10\ \text{m})(5.0\ \text{m/s}) = 0.60\ \text{V}\ .$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is $i = 0.60\ \text{V}/0.40\ \Omega = 1.5\ \text{A}$.

(d) The direction is clockwise.

(e) Equation 26-28 leads to $P = i^2R = 0.90$ W.

$$B = \mu_0 n \Delta i = \mu_0 \left(\frac{N}{W}\right)\left(\frac{i}{N}\right) = \frac{\mu_0 i}{W} = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.035\mathrm{A})}{0.16\mathrm{m}} = 2.7\times10^{-7}\,\mathrm{T}.$$

(b) Equation 30-33 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2 (\mu_0 i/W)}{i} = \frac{\pi\mu_0 R^2}{W} = \frac{\pi(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(0.018\mathrm{m})^2}{0.16\mathrm{m}} = 8.0\times10^{-9}\,\mathrm{H}.$$

43. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B/ℓ. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B(\ell\,dx) + 2\int_a^{d/2} B(\ell\,dx)$$

where d = 0.0025 m is the diameter of each wire. We will use r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2} r + \frac{\mu_0 i}{2\pi(d-r)}\right) dr + 2\int_a^{d/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi \ln((d-a)/a)$. Now, we use Eq. 30-33 (with $N = 1$) to obtain the inductance per unit length:

$$\frac{L}{\ell} = \frac{\Phi_B}{\ell i} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = 1.81\times10^{-6}\,\mathrm{H/m}.$$

44. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\varepsilon}{L} = -\frac{60\mathrm{V}}{12\mathrm{H}} = -5.0\,\mathrm{A/s},$$