13. **THINK** As defined in Eq. 32-38, magnetization is the dipole moment per unit volume.

EXPRESS Let M be the magnetization and $\mathcal{V}$ be the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). The dipole moment is given by $\mu = M\mathcal{V}$.

ANALYZE Substituting the values given, we obtain

$$\mu = M\pi r^2 L = \left(5.30\times10^3 \text{ A/m}\right)\pi\left(3.00\times10^{-3}\text{ m}\right)^2\left(5.00\times10^{-2}\text{ m}\right) = 7.49\times10^{-2}\text{ J/T}\ .$$

LEARN In a sample with N atoms, the magnetization reaches maximum, or saturation, when all the dipoles are completely aligned, leading to $M_{\text{max}} = N\mu/\mathcal{V}$.

14. (a) From Eq. 32-10,

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}\,\varepsilon_0 A \frac{d}{dt}\left[\left(4.0\times10^5\right)-\left(6.0\times10^4 t\right)\right] = -\varepsilon_0 A\left(6.0\times10^4\text{ V/m}\cdot\text{s}\right) \\ &= -\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(6.0\times10^{-2}\text{m}^2\right)\left(6.0\times10^4\text{ V/m}\cdot\text{s}\right) \\ &= -3.2\times10^{-8}\text{A}. \end{aligned}$$

Thus, the magnitude of the displacement current is $|i_d| = 3.2\times10^{-8}\text{A}$.

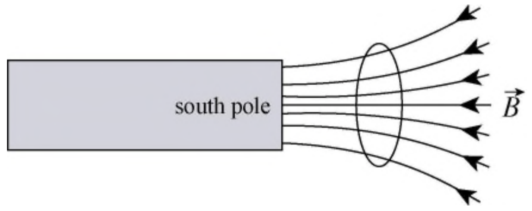
(b) The negative sign in i_d implies that the direction is downward.

(c) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-18,

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

19. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of Section 32-9 is two-fold: $\vec{u}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material toward regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch, or in the $+x$ direction.

(c) The direction of the current is clockwise (from the perspective of the bar magnet).

(d) Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is toward the right in our sketch, or in the $+x$ direction.

30. Combining Eq. 32-27 with Eqs. 32-22 and 32-23, we see that the energy difference is

$$\Delta U = 2\mu_B B$$

where μ_B is the Bohr magneton (given in Eq. 32-25). With ΔU = 4.00 × 10^{-25} J, we obtain B = 2.16 mT.

39. **THINK** In this problem, we model the Earth's magnetic dipole moment with a magnetized iron sphere.

EXPRESS If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}.$$

We substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3\mu}{3m} \quad\Rightarrow\quad R = \left(\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right)^{1/3}.$$

ANALYZE (a) The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\ \text{kg/u}) = 9.30\times10^{-26}\ \text{kg}.$$

Therefore, the radius of the iron sphere is

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\ \text{J/T})}{4\pi(14\times10^{3}\ \text{kg/m}^3)(2.1\times10^{-23}\ \text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}.$$

(b) The volume of the sphere is $V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82\times10^5\text{m})^3 = 2.53\times10^{16}\,\text{m}^3$ and the volume of the Earth is

$$V_E = \frac{4\pi}{3}R_E^3 = \frac{4\pi}{3}\left(6.37\times10^6\,\text{m}\right)^3 = 1.08\times10^{21}\,\text{m}^3,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{V_s}{V_E} = \frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}.$$

LEARN The finding that $V_s \ll V_E$ makes it unlikely that our simple model of a magnetized iron sphere could explain the origin of Earth's magnetization.

Chapter 32

1. We use $\sum_{n=1}^{6}\Phi_{Bn}=0$ to obtain

$$\Phi_{B6}=-\sum_{n=1}^{5}\Phi_{Bn}=-\left(-1\,\text{Wb}+2\,\text{Wb}-3\,\text{Wb}+4\,\text{Wb}-5\,\text{Wb}\right)=+3\,\text{Wb}\ .$$

2. (a) The flux through the top is $+(0.30\text{ T})\pi r^2$ where $r = 0.020$ m. The flux through the bottom is +0.70 mWb as given in the problem statement. Since the *net* flux must be zero then the flux through the sides must be negative and exactly cancel the total of the previously mentioned fluxes. Thus (in magnitude) the flux though the sides is 1.1 mWb.

(b) The fact that it is negative means it is inward.

3. (a) We use Gauss' law for magnetism: $\oint \vec{B}\cdot d\vec{A}=0$. Now,

$$\oint \vec{B}\cdot d\vec{A}=\Phi_1+\Phi_2+\Phi_C,$$

where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\ \mu$Wb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. Its value is

$$\Phi_2=\pi(0.120\,\text{m})^2\left(1.60\times10^{-3}\,\text{T}\right)=+7.24\times10^{-5}\,\text{Wb}=+72.4\,\mu\text{Wb}\ .$$

Since the three fluxes must sum to zero,

$$\Phi_C=-\Phi_1-\Phi_2=25.0\,\mu\text{Wb}-72.4\,\mu\text{Wb}=-47.4\,\mu\text{Wb}\ .$$

Thus, the magnitude is $|\Phi_C|=47.4\,\mu\text{Wb}$.

(b) The minus sign in Φ_C indicates that the flux is inward through the curved surface.

4. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\Phi_B(S_1)=\Phi_B(S_2)=\int_{-r}^{r}B(x)\,L\,dx=2\int_{-r}^{r}B_{\text{left}}(x)\,L\,dx=2\int_{-r}^{r}\frac{\mu_0 i}{2\pi}\frac{1}{2r-x}\,L\,dx=\frac{\mu_0 iL}{\pi}\ln 3\ .$$

5. We use the result of part (b) in Sample Problem — "Magnetic field induced by changing electric field,"

$$B=\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt},\qquad (r\geq R)$$

to solve for dE/dt:

$$\frac{dE}{dt}=\frac{2Br}{\mu_0\varepsilon_0 R^2}=\frac{2\left(2.0\times10^{-7}\ \text{T}\right)\left(6.0\times10^{-3}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.0\times10^{-3}\text{m}\right)^2}=2.4\times10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}}.$$

6. The integral of the field along the indicated path is, by Eq. 32-18 and Eq. 32-19, equal to

$$\mu_0 i_d\left(\frac{\text{enclosed area}}{\text{total area}}\right)=\mu_0\,(0.75\ \text{A})\,\frac{(4.0\ \text{cm})(2.0\ \text{cm})}{12\ \text{cm}^2}=52\ \text{nT}\cdot\text{m}\ .$$

7. (a) Inside we have (by Eq. 32-16) $B=\mu_0 i_d r_1/2\pi R^2$, where $r_1=0.0200$ m, $R=0.0300$ m, and the displacement current is given by Eq. 32-38 (in SI units):

$$i_d=\varepsilon_0\frac{d\Phi_E}{dt}=(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{-3}\ \text{V/m}\cdot\text{s})=2.66\times10^{-14}\ \text{A}\ .$$

Thus we find

$$B=\frac{\mu_0 i_d r_1}{2\pi R^2}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})(0.0200\ \text{m})}{2\pi(0.0300\ \text{m})^2}=1.18\times10^{-19}\ \text{T}\ .$$

(b) Outside we have (by Eq. 32-17) $B=\mu_0 i_d/2\pi r_2$ where r_2 = 0.0500 cm. Here we obtain

$$B=\frac{\mu_0 i_d}{2\pi r_2}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.66\times10^{-14}\ \text{A})}{2\pi(0.0500\ \text{m})}=1.06\times10^{-19}\ \text{T}$$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

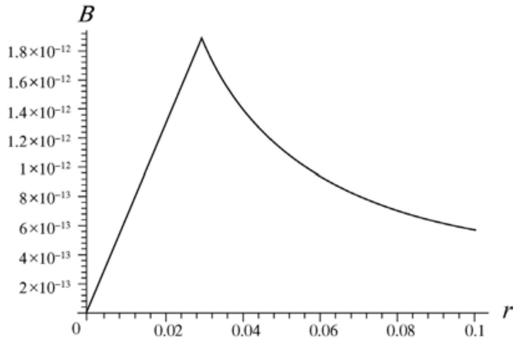
$$B(2\pi r)=\varepsilon_0\mu_0\left(0.60\ \text{V}\cdot\text{m/s}\right)\frac{r}{R}.$$

Using r = 0.0200 m (which, in any case, cancels out) and R = 0.0300 m, we obtain

$$B_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} = \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}V_{\max}\omega\cos(\omega t)\right)_{\max}$$

$$= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \quad (\text{for } r \geq R)$$

(note the $B \propto r^{-1}$ dependence — see also Eqs. 32-16 and 32-17). The plot (with SI units understood) is shown below.



12. From Sample Problem — "Magnetic field induced by changing electric field," we know that $B \propto r$ for $r \leq R$ and $B \propto r^{-1}$ for $r \geq R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of $B_{\max}$. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$.

(a) Inside the capacitor, $0.75\, B_{\max}/B_{\max} = r_1/R$, or $r_1 = 0.75\, R = 0.75$ (40 mm) =30 mm.

(b) Outside the capacitor, $0.75\, B_{\max}/B_{\max} = (r_2/R)^{-1}$, or

$$r_2 = R/0.75 = 4R/3 = (4/3)(40 \text{ mm}) = 53 \text{ mm}.$$

(c) From Eqs. 32-15 and 32-17,

$$B_{\max} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(6.0\ \text{A})}{2\pi(0.040\ \text{m})} = 3.0\times10^{-5}\ \text{T}.$$

13. Let the area plate be A and the plate separation be d. We use Eq. 32-10:

$$i_d = \varepsilon_0\frac{d\Phi_E}{dt} = \varepsilon_0\frac{d}{dt}(AE) = \varepsilon_0 A\frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right),$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\,\text{A}}{2.0\times10^{-6}\ \text{F}} = 7.5\times10^5\ \text{V/s}.$$