45. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(1.40\times 10^{3}\ \mathrm{W/m^2}\right)\left(2.998\times 10^{8}\ \mathrm{m/s}\right)}$$
$$= 1.03\times 10^{3}\ \mathrm{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03\times 10^{4}\ \mathrm{V/m}}{2.998\times 10^{8}\ \mathrm{m/s}} = 3.43\times 10^{-6}\ \mathrm{T}.$$

(c) The rms value of the electric field is

$$E_{\mathrm{rms}} = \frac{E_m}{\sqrt{2}} = \frac{1.03\times 10^{3}\ \mathrm{V/m}}{\sqrt{2}} = 726\ \mathrm{V/m}.$$

(d) Similarly, the rms value of the magnetic field is

$$B_{\mathrm{rms}} = \frac{B_m}{\sqrt{2}} = \frac{3.43\times 10^{-6}\ \mathrm{T}}{\sqrt{2}} = 2.42\times 10^{-6}\ \mathrm{T}.$$

$$\left(\frac{\partial B}{\partial t}\right)_{\max} = \sqrt{\frac{2\mu_0 P}{4\pi c}}\frac{2\pi c}{\lambda r} = 3.44\times 10^6 \text{ T/s}.$$

15. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2\left(4\pi\times 10^{-7}\text{ H/m}\right)\left(2.998\times 10^8\text{ m/s}\right)\left(10\times 10^{-6}\text{ W/m}^2\right)}$$
$$= 8.7\times 10^{-2}\text{ V/m}.$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7\times 10^{-2}\text{ V/m}}{2.998\times 10^8\text{ m/s}} = 2.9\times 10^{-10}\text{ T}.$$

(c) At a distance r from the transmitter, the intensity is $I = P/2\pi r^2$, where P is the power of the transmitter over the hemisphere having a surface area $2\pi r^2$. Thus

$$P = 2\pi r^2 I = 2\pi\left(10\times 10^3\text{ m}\right)^2\left(10\times 10^{-6}\text{ W/m}^2\right) = 6.3\times 10^3\text{ W}.$$

16. (a) The power received is

$$P_r = \left(1.0\times 10^{-12}\text{ W}\right)\frac{\pi(300\text{ m})^2/4}{4\pi\left(6.37\times 10^6\text{ m}\right)^2} = 1.4\times 10^{-22}\text{ W}.$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi\left[\left(2.2\times 10^4\text{ ly}\right)\left(9.46\times 10^{15}\text{ m/ly}\right)\right]^2\left[\frac{1.0\times 10^{-12}\text{ W}}{4\pi\left(6.37\times 10^6\text{ m}\right)^2}\right] = 1.1\times 10^{15}\text{ W}.$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\text{ V/m}}{2.998\times 10^8\text{ m/s}} = 6.7\times 10^{-9}\text{ T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\text{ V/m})^2}{2\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(2.998\times 10^8\text{ m/s}\right)} = 5.3\times 10^{-3}\text{ W/m}^2.$$

wave. The electromagnetic energy inside is $U = uA\ell,$ where u is the energy density. All this energy passes through the end in time $t = \ell / c,$ so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc.$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is

$$I = I_0 + f I_0 = (1 + f) I_0,$$

where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f) I_0}{c},$$

the same as radiation pressure.

26. The mass of the cylinder is $m = \rho(\pi D^2/4)H,$ where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g\rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$H = \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho}$$

$$= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\text{ m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\text{ m/s})(1.20\times10^3\,\text{kg/m}^3)}$$

$$= 4.91\times10^{-7}\text{ m}.$$

27. (a) Since $c = \lambda f,$ where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{2.998\times10^8\text{ m/s}}{3.0\text{ m}} = 1.0\times10^8\text{ Hz}.$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0\times10^8\text{ Hz}) = 6.3\times10^8\text{ rad/s}.$$

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\ \text{m}} = 2.1\ \text{rad / m}.$$

(d) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\text{V/m}}{2.998\times10^8\,\text{m/s}} = 1.0\times10^{-6}\,\text{T}.$$

(e) $\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E}\times\vec{B}$ to be in the positive x direction (the direction of propagation).

(f) The intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{H/m})(2.998\times10^8\,\text{m/s})} = 119\,\text{W/m}^2 \approx 1.2\times10^2\,\text{W/m}^2.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\ \text{W / m}^2)(2.0\ \text{m}^2)}{2.998\times10^8\ \text{m / s}} = 8.0\times10^{-7}\ \text{N}.$$

(h) The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0\times10^{-7}\ \text{N}}{2.0\ \text{m}^2} = 4.0\times10^{-7}\ \text{Pa}.$$

28. (a) Assuming complete absorption, the radiation pressure is

$$p_r = \frac{I}{c} = \frac{1.4\times10^3\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}} = 4.7\times10^{-6}\ \text{N/m}^2.$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0} = \frac{4.7\times10^{-6}\ \text{N/m}^2}{1.0\times10^5\ \text{N/m}^2} = 4.7\times10^{-11}.$$

29. If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$F_g = \frac{GM_S m}{r^2} = \frac{GM_S \rho(4\pi R^3/3)}{r^2} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

where $m = \rho(4\pi R^3/3)$ is the mass of the particle. When the two forces balance, the particle travels in a straight path. The condition that $F_r = F_g$ implies

$$\frac{P_S R^2}{4r^2 c} = \frac{4\pi GM_S \rho R^3}{3r^2},$$

which can be solved to give

$$R = \frac{3P_S}{16\pi c\rho GM_S} = \frac{3(3.9\times10^{26}\ \text{W})}{16\pi(3\times10^8\ \text{m/s})(3.5\times10^3\ \text{kg/m}^3)(6.67\times10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2)(1.99\times10^{30}\ \text{kg})}$$
$$= 1.7\times10^{-7}\ \text{m}.$$

(b) Since F_g varies with R^3 and F_r varies with R^2, if the radius R is larger, then $F_g > F_r$, and the path will be curved toward the Sun (like path 3).

32. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1+\theta_2)\cos^2(\theta_2+\theta_3) = \frac{1}{2}\cos^2(50°+50°)\cos^2(50°+50°)$$
$$= 4.5\times10^{-4}.$$

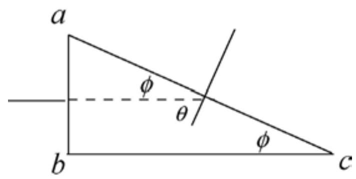
Thus, 0.045% of the light's initial intensity is transmitted.

33. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is θ_1 = 40° counterclockwise from the y axis in the diagram. The polarizing direction of the second sheet is θ_2 = 20° clockwise from the y axis, so the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet is 40° + 20° = 60°. The transmitted intensity is

$$I_2 = I_1\cos^2 60° = \frac{1}{2}I_0\cos^2 60°,$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis. The polarizing direction of the third sheet is θ_3 = 40° counterclockwise from the y axis. Consequently, the angle between the direction of polarization of the light incident on that

59. (a) No refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $90° - \phi$, as shown in the figure below.



For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

60. (a) The condition (in Eq. 33-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Yes. Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

(c) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 51.1°$.

(d) No. Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

61. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leading to $\theta = 26.8°$.

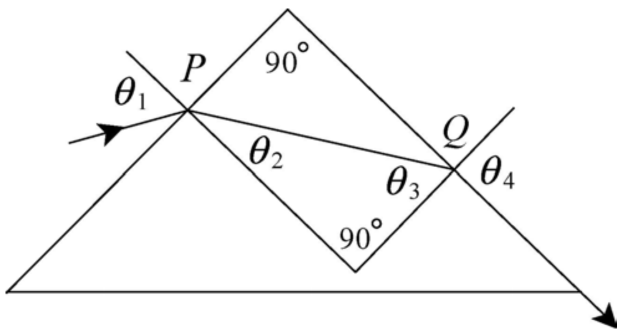
(b) Increasing θ leads to a decrease of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle; therefore, there will be some transmission of light into material 3.

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan\theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin\theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00 \text{ m})(\tan 48.75°) = 4.56 \text{ m}.$$

(b) The diameter d of the circle will increase if the fish descends (increasing h).

63. (a) A ray diagram is shown below.



Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is $\theta_4 = 90°$. The law of refraction, applied to the second surface, yields $n \sin\theta_3 = \sin\theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and

$$\sin\theta_1 = n\sin\theta_2 = 1.60 \sin 21.32° = 0.5817.$$

Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point *C*. Since the angle of refraction there is the same as the angle of incidence at *A*, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin(60° - \theta_2) \Rightarrow \sin\theta_1 = n\sin 60° \cos\theta_2 - n\cos 60° \sin\theta_2$$

where the trigonometric identity

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

is used. Next, we apply the law of refraction to point *A*:

$$\sin\theta_1 = n\sin\theta_2 \Rightarrow \sin\theta_2 = (1/n)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-(1/n^2)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60° \sqrt{1-(1/n)^2\sin^2\theta_1} - \cos 60° \sin\theta_1$$

or

$$(1+\cos 60°)\sin\theta_1 = \sin 60° \sqrt{n^2 - \sin^2\theta_1}.$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = 0.80$$

and $\theta_1 = 53.1°$.

65. When examining Fig. 33-61, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta$$

assuming $n_{\text{air}} = 1$. The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp}, and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1-\sin^2\theta'}.$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1-\sin^2\theta'} = \sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$. With n_1 = 1.58 and n_2 = 1.53, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2°.$$

66. (a) We note that the upper-right corner is at an angle (measured from the point where the light enters, and measured relative to a normal axis established at that point the normal at that point would be horizontal in Fig. 33-62) is at $\tan^{-1}(2/3)$ = 33.7°. The angle of refraction is given by

$$n_{\text{air}} \sin 40° = 1.56 \sin \theta_2$$

which yields θ_2 = 24.33° if we use the common approximation n_{air} = 1.0, and yields θ_2 = 24.34° if we use the more accurate value for n_{air} found in Table 33-1. The value is less than 33.7°, which means that the light goes to side 3.

(b) The ray strikes a point on side 3, which is 0.643 cm below that upper-right corner, and then (using the fact that the angle is symmetrical upon reflection) strikes the top surface (side 2) at a point 1.42 cm to the left of that corner. Since 1.42 cm is certainly less than 3 cm we have a self-consistency check to the effect that the ray does indeed strike side 2 as its second reflection (if we had gotten 3.42 cm instead of 1.42 cm, then the situation would be quite different).

(c) The normal axes for sides 1 and 3 are both horizontal, so the angle of incidence (in the plastic) at side 3 is the same as the angle of refraction was at side 1. Thus,

$$1.56 \sin 24.3° = n_{\text{air}} \sin \theta_{\text{air}} \quad \Rightarrow \quad \theta_{\text{air}} = 40° .$$

(d) It strikes the top surface (side 2) at an angle (measured from the normal axis there, which in this case would be a vertical axis) of 90° $-\ \theta_2$ = 66°, which is much greater than the critical angle for total internal reflection ($\sin^{-1}(n_{\text{air}}/1.56)$ = 39.9°). Therefore, no refraction occurs when the light strikes side 2.

(e) In this case, we have

$$n_{\text{air}} \sin 70° = 1.56 \sin \theta_2$$

which yields θ_2 = 37.04° if we use the common approximation n_{air} = 1.0, and yields θ_2 = 37.05° if we use the more accurate value for n_{air} found in Table 33-1. This is greater than