

24. In this setup, we have  $n_2 < n_1$  and  $n_2 > n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow L = \left(m + \frac{1}{2}\right) \frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \dots$$

The second least thickness is ( $m = 1$ )

$$L = \left(1 + \frac{1}{2}\right) \frac{342 \text{ nm}}{2(1.59)} = 161 \text{ nm}.$$

25. **THINK** The formation of Newton's rings is due to the interference between the rays reflected from the flat glass plate and the curved lens surface.

**EXPRESS** Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a  $\pi$  rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is  $d$ , the condition for a maximum in intensity is  $2d = (m + \frac{1}{2})\lambda$ , where  $\lambda$  is the wavelength in air and  $m$  is an integer. Therefore,

$$d = (2m + 1)\lambda/4.$$

**ANALYZE** As the geometry of Fig. 35-46 shows,  $d = R - \sqrt{R^2 - r^2}$ , where  $R$  is the radius of curvature of the lens and  $r$  is the radius of a Newton's ring. Thus,  $(2m + 1)\lambda/4 = R - \sqrt{R^2 - r^2}$ . First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m + 1)\lambda}{4}.$$

Next, we square both sides, rearrange to solve for  $r^2$ , then take the square root. We get

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2} - \frac{(2m + 1)^2 \lambda^2}{16}}.$$

If  $R$  is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2}}.$$

**LEARN** Similarly, the radii of the dark fringes are given by  $r = \sqrt{mR\lambda}$ .

**33. (a)** We choose a horizontal  $x$  axis with its origin at the left edge of the plastic. Between  $x = 0$  and  $x = L_2$  the phase difference is that given by Eq. 35-11 (with  $L$  in that equation replaced with  $L_2$ ). Between  $x = L_2$  and  $x = L_1$  the phase difference is given by an expression similar to Eq. 35-11 but with  $L$  replaced with  $L_1 - L_2$  and  $n_2$  replaced with 1 (since the top ray in Fig. 35-36 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences with  $\lambda = 0.600 \mu\text{m}$ , we have

$$\begin{aligned} \frac{L_2}{\lambda}(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}(1 - n_1) &= \frac{3.50 \mu\text{m}}{0.600 \mu\text{m}}(1.60 - 1.42) + \frac{4.00 \mu\text{m} - 3.50 \mu\text{m}}{0.600 \mu\text{m}}(1 - 1.42) \\ &= 1.05 + (-0.35) = 0.70. \end{aligned}$$

(b) The answer in part (a) is intermediate but closer to a half-integer, so the interference is more nearly destructive than constructive.

41. The maxima of a two-slit interference pattern are at angles  $\theta$  given by  $d \sin \theta = m\lambda$ , where  $d$  is the slit separation,  $\lambda$  is the wavelength, and  $m$  is an integer. If  $\theta$  is small,  $\sin \theta$  may be replaced by  $\theta$  in radians. Then,  $d\theta = m\lambda$ . The angular separation of two maxima associated with different wavelengths but the same value of  $m$  is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance  $D$  away is

$$\begin{aligned}\Delta y &= D \tan \Delta\theta \approx D \Delta\theta = \left[ \frac{mD}{d} \right] (\lambda_2 - \lambda_1) \\ &= \left[ \frac{3(1.8 \text{ m})}{5.0 \times 10^{-3} \text{ m}} \right] (600 \times 10^{-9} \text{ m} - 480 \times 10^{-9} \text{ m}) = 1.3 \times 10^{-4} \text{ m}.\end{aligned}$$

The small angle approximation  $\tan \Delta\theta \approx \Delta\theta$  (in radians) is made.

$$y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4 \sin(\omega t + 13.3^\circ).$$

Quoting the answer to two significant figures, we have  $y \approx 17 \sin(\omega t + 13^\circ)$ .

31. In adding these with the phasor method (as opposed to, say, trig identities), we may set  $t = 0$  and add them as vectors:

$$y_h = 10 \cos 0^\circ + 15 \cos 30^\circ + 5.0 \cos(-45^\circ) = 26.5$$

$$y_v = 10 \sin 0^\circ + 15 \sin 30^\circ + 5.0 \sin(-45^\circ) = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 26.8 \approx 27$$

$$\beta = \tan^{-1} \left( \frac{y_v}{y_h} \right) = 8.5^\circ.$$

Thus,  $y = y_1 + y_2 + y_3 = y_R \sin(\omega t + \beta) = 27 \sin(\omega t + 8.5^\circ)$ .

32. (a) We can use phasor techniques or use trig identities. Here we show the latter approach. Since

$$\sin a + \sin(a + b) = 2 \cos(b/2) \sin(a + b/2),$$

we find

$$E_1 + E_2 = 2E_0 \cos(\phi/2) \sin(\omega t + \phi/2)$$

where  $E_0 = 2.00 \mu\text{V/m}$ ,  $\omega = 1.26 \times 10^{15} \text{ rad/s}$ , and  $\phi = 39.6 \text{ rad}$ . This shows that the electric field amplitude of the resultant wave is

$$E = 2E_0 \cos(\phi/2) = 2(2.00 \mu\text{V/m}) \cos(19.2 \text{ rad}) = 2.33 \mu\text{V/m}.$$

(b) Equation 35-22 leads to

$$I = 4I_0 \cos^2(\phi/2) = 1.35 I_0$$

at point  $P$ , and

$$I_{\text{center}} = 4I_0 \cos^2(0) = 4 I_0$$

at the center. Thus,  $I/I_{\text{center}} = 1.35/4 = 0.338$ .

(c) The phase difference  $\phi$  (in wavelengths) is gotten from  $\phi$  in radians by dividing by  $2\pi$ . Thus,  $\phi = 39.6/2\pi = 6.3$  wavelengths. Thus, point  $P$  is between the sixth side maximum (at which  $\phi = 6$  wavelengths) and the seventh minimum (at which  $\phi = 6\frac{1}{2}$  wavelengths).

(d) The rate is given by  $\omega = 1.26 \times 10^{15}$  rad/s.

(e) The angle between the phasors is  $\phi = 39.6$  rad =  $2270^\circ$  (which would look like about  $110^\circ$  when drawn in the usual way).

33. With phasor techniques, this amounts to a vector addition problem  $\vec{R} = \vec{A} + \vec{B} + \vec{C}$  where (in magnitude-angle notation)  $\vec{A} = (10 \angle 0^\circ)$ ,  $\vec{B} = (5 \angle 45^\circ)$ , and  $\vec{C} = (5 \angle -45^\circ)$ , where the magnitudes are understood to be in  $\mu\text{V/m}$ . We obtain the resultant (especially efficient on a vector-capable calculator in polar mode):

$$\vec{R} = (10 \angle 0^\circ) + (5 \angle 45^\circ) + (5 \angle -45^\circ) = (17.1 \angle 0^\circ)$$

which leads to

$$E_R = (17.1 \mu\text{V/m}) \sin(\omega t)$$

where  $\omega = 2.0 \times 10^{14}$  rad/s.

34. (a) Referring to Figure 35-10(a) makes clear that

$$\theta = \tan^{-1}(y/D) = \tan^{-1}(0.205/4) = 2.93^\circ.$$

Thus, the phase difference at point  $P$  is  $\phi = d \sin \theta / \lambda = 0.397$  wavelengths, which means it is between the central maximum (zero wavelength difference) and the first minimum ( $\frac{1}{2}$  wavelength difference). Note that the above computation could have been simplified somewhat by avoiding the explicit use of the tangent and sine functions and making use of the small-angle approximation ( $\tan \theta \approx \sin \theta$ ).

(b) From Eq. 35-22, we get (with  $\phi = (0.397)(2\pi) = 2.495$  rad)

$$I = 4I_0 \cos^2(\phi/2) = 0.404 I_0$$

at point  $P$  and

$$I_{\text{center}} = 4I_0 \cos^2(0) = 4 I_0$$

at the center. Thus,  $I/I_{\text{center}} = 0.404/4 = 0.101$ .

35. For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of  $\pi$  rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of  $\pi$  rad on reflection. If  $L$  is the thickness of the coating, the wave reflected from the back surface travels a distance  $2L$  farther than the wave reflected from the front. The phase difference is  $2L(2\pi/\lambda_c)$ , where  $\lambda_c$  is the wavelength in the coating. If  $n$  is the index of refraction of the coating,  $\lambda_c = \lambda/n$ , where  $\lambda$  is the wavelength in vacuum, and the phase difference is  $2nL(2\pi/\lambda)$ . We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for  $L$ . Here  $m$  is an integer. The result is

$$L = \frac{(2m+1)\lambda}{4n}.$$

To find the least thickness for which destructive interference occurs, we take  $m = 0$ . Then,

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9} \text{ m}}{4(1.25)} = 1.20 \times 10^{-7} \text{ m}.$$

36. (a) On both sides of the soap is a medium with lower index (air) and we are examining the reflected light, so the condition for strong reflection is Eq. 35-36. With lengths in nm,

$$\lambda = \frac{2n_2L}{m + \frac{1}{2}} = \begin{cases} 3360 & \text{for } m = 0 \\ 1120 & \text{for } m = 1 \\ 672 & \text{for } m = 2 \\ 480 & \text{for } m = 3 \\ 373 & \text{for } m = 4 \\ 305 & \text{for } m = 5 \end{cases}$$

from which we see the latter *four* values are in the given range.

(b) We now turn to Eq. 35-37 and obtain

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1680 & \text{for } m = 1 \\ 840 & \text{for } m = 2 \\ 560 & \text{for } m = 3 \\ 420 & \text{for } m = 4 \\ 336 & \text{for } m = 5 \end{cases}$$

from which we see the latter *three* values are in the given range.

37. Light reflected from the front surface of the coating suffers a phase change of  $\pi$  rad while light reflected from the back surface does not change phase. If  $L$  is the thickness of the coating, light reflected from the back surface travels a distance  $2L$  farther than light reflected from the front surface. The difference in phase of the two waves is  $2L(2\pi/\lambda_c) - \pi$ , where  $\lambda_c$  is the wavelength in the coating. If  $\lambda$  is the wavelength in vacuum, then  $\lambda_c = \lambda/n$ , where  $n$  is the index of refraction of the coating. Thus, the phase difference is  $2nL(2\pi/\lambda) - \pi$ . For fully constructive interference, this should be a multiple of  $2\pi$ . We solve

61. In this setup, we have  $n_2 > n_1$  and  $n_2 > n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \dots$$

Therefore,

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} & (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 758 \text{ nm} & (m=1) \\ 4Ln_2/5 = 4(415 \text{ nm})(1.59)/5 = 455 \text{ nm} & (m=2) \end{cases}$$

For the wavelength to be in the visible range, we choose  $m = 2$  with  $\lambda = 455 \text{ nm}$ .

62. In this setup, we have  $n_2 < n_1$  and  $n_2 > n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow L = \left(m + \frac{1}{2}\right) \frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \dots$$

The second least thickness is ( $m = 1$ )

$$L = \left(1 + \frac{1}{2}\right) \frac{342 \text{ nm}}{2(1.59)} = 161 \text{ nm}.$$

63. In this setup, we have  $n_2 > n_1$  and  $n_2 < n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow L = \left(m + \frac{1}{2}\right) \frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \dots$$

The second least thickness is ( $m = 1$ )

$$L = \left(1 + \frac{1}{2}\right) \frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm}.$$

64. In this setup, we have  $n_2 > n_1$  and  $n_2 < n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right) \frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \dots$$

Thus, we have