9. Assuming all $N = 1600$ lines are uniformly illuminated, we have

$$\frac{\lambda_{av}}{\Delta\lambda} = Nm$$

from Eq. 36-31 and Eq. 36-32. With $\lambda_{av} = 600$ nm and $m = 2$, we find

$$\Delta\lambda = \frac{\lambda_{av}}{Nm} = \frac{600 \text{ nm}}{(1600)(2)} = 0.19 \text{ nm}.$$

13. **THINK** We apply the Rayleigh criterion to estimate the linear separation between the two objects.

EXPRESS If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians.

ANALYZE (a) With small angle approximation, $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. Thus,

$$D = \frac{1.22\,L\lambda}{d} = \frac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.0\times10^{-3}\text{ m}} = 1.1\times10^{7}\text{ m} = 1.1\times10^{4}\text{ km}\,.$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

(b) Now d = 5.1 m and $D = \dfrac{1.22\left(8.0\times10^{10}\text{ m}\right)\left(550\times10^{-9}\text{ m}\right)}{5.1\text{m}} = 1.1\times10^{4}\text{ m} = 11\text{ km}$.

LEARN By the Rayleigh criterion for resolvability, two objects can be resolved only if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$.

29. **THINK** For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity; instead, the intensities are modified by diffraction of light passing through each slit.

EXPRESS The angular positions θ of the bright interference fringes are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a \sin \theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which

$$-\sin \theta_1 < \sin \theta < +\sin \theta_1 .$$

The intensity at the screen is given by

$$I = I_m \left(\cos^2 \beta\right)\left(\frac{\sin \alpha}{\alpha}\right)^2$$

where $\alpha = (\pi a/\lambda) \sin \theta$, $\beta = (\pi d/\lambda) \sin \theta$, and I_m is the intensity at the center of the pattern.

ANALYZE (a) The condition above means $-1/a < m/d < 1/a$, or $-d/a < m < +d/a$. Now

$$d/a = (0.180 \times 10^{-3}\ \text{m})/(30.0 \times 10^{-6}\ \text{m}) = 6.00,$$

so the values of m are m = –5, –4, –3, –2, –1, 0, +1, +2, +3, +4, and +5. There are 11 fringes.

(b) For the third bright interference fringe, $d \sin \theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2 \beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/6.00 = 0.500\pi$ rad and

$$\left(\frac{\sin \alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.500\pi}{0.500\pi}\right)^2 = 0.405 .$$

The intensity ratio is $I/I_m = 0.405$.

LEARN The expression for intensity contains two factors: (1) the interference factor $\cos^2 \beta$ due to the interference between two slits with separation d, and (2) the diffraction factor $[(\sin \alpha)/\alpha]^2$ which arises due to diffraction by a single slit of width a. In the limit $a \to 0$, $(\sin \alpha)/\alpha \to 1$, and we recover Eq. 35-22 for the interference between two slits of vanishingly narrow slits separated by d. Similarly, setting $d = 0$ or equivalently, $\beta = 0$, we recover Eq. 36-5 for the diffraction of a single slit of width a.

49. **THINK** The relative intensity in a single-slit diffraction depends on the ratio a/λ, where a is the slit width and λ is the wavelength.

EXPRESS The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2 \alpha}{\alpha^2}$$

where I_m is the maximum intensity and $\alpha = (\pi a/\lambda) \sin \theta$. The angle θ is measured from the forward direction.

ANALYZE (a) We require $I = I_m/2$, so

$$\sin^2 \alpha = \frac{1}{2}\alpha^2 \ .$$

(b) We evaluate $\sin^2 \alpha$ and $\alpha^2/2$ for α = 1.39 rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda) \sin \theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right).$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

(d) For a/λ = 2.0, $\Delta\theta = 2\sin^{-1}(0.442/2.0) = 25.5°$.

(e) For a/λ = 7.0, $\Delta\theta = 2\sin^{-1}(0.442/7.0) = 7.24°$.

(f) For a/λ = 12, $\Delta\theta = 2\sin^{-1}(0.442/12) = 4.22°$.

LEARN The wider the slit is (relative to the wavelength), the narrower is the central diffraction maximum.

64. (a) Using the notation of Sample Problem — "Pointillistic paintings use the diffraction of your eye," the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{(420\times 10^3\,\text{m})(1.22)(550\times 10^{-9}\,\text{m})}{(0.005\,\text{m})} \approx 56\ \text{m}.$$

(b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

(c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

69. The condition for a minimum of a single-slit diffraction pattern is $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is 1.00° for m = 1. Thus,

$$a = \frac{m\lambda}{\sin\theta} = \frac{420\times10^{-9}\text{ m}}{\sin 1.00°} = 2.41\times10^{-5}\text{ m}.$$