Chapter 38

1. (a) We assume all the power results in photon production at the wavelength $\lambda = 589$ nm. Let R be the rate of photon production and E be the energy of a single photon. Then,

$$P = RE = Rhc/\lambda,$$

where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{(589\times10^{-9}\text{ m})(100\text{ W})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})} = 2.96\times10^{20}\text{ photon/s}.$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96\times10^{20}\text{ photon/s}}{4\pi(1.00\times10^{4}\text{ photon/m}^2\cdot\text{s})}} = 4.86\times10^{7}\text{ m}.$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96\times10^{20}\text{ photon/s}}{4\pi(2.00\text{ m})^2} = 5.89\times10^{18}\,\frac{\text{photon}}{\text{m}^2\cdot\text{s}}.$$

11. We use the uncertainty relationship $\Delta x \Delta p \geq \hbar$. Letting $\Delta x = \lambda$, the de Broglie wavelength, we solve for the minimum uncertainty in p:

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\lambda} = \frac{p}{2\pi}$$

where the de Broglie relationship $p = h/\lambda$ is used. We use $1/2\pi = 0.080$ to obtain $\Delta p = 0.080p$. We would expect the measured value of the momentum to lie between $0.92p$ and $1.08p$. Measured values of zero, $0.5p$, and $2p$ would all be surprising.

16. (a) The momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}} = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$$

(b) The momentum of the photon is the same as that of the electron: $p = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

(c) The kinetic energy of the electron is

$$K_e = \frac{p^2}{2m_e} = \frac{\left(3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)} = 6.0\times10^{-18}\ \text{J} = 38\ \text{eV}.$$

(d) The kinetic energy of the photon is

$$K_{\text{ph}} = pc = \left(3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)\left(2.998\times10^{8}\ \text{m/s}\right) = 9.9\times10^{-16}\ \text{J} = 6.2\ \text{keV}.$$

17. THINK **In this problem we solve a special case of the Schrödinger's equation where the potential energy is** $U(x) = U_0 =$ constant.

EXPRESS For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

We substitute $\psi = \psi_0 e^{ikx}$.

ANALYZE The second derivative is $\frac{d^2\psi}{dx^2} = -k^2\psi_0 e^{ikx} = -k^2\psi$. The result is

$$-k^2\psi + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

Solving for k, we obtain

$$k = \sqrt{\frac{8\pi^2 m}{h^2}[E - U_0]} = \frac{2\pi}{h}\sqrt{2m[E - U_0]}.$$

LEARN Another way to realize this is to note that with a constant potential energy $U(x) = U_0$, we can simply redefine the total energy as $E' = E - U_0$, and the Schrödinger's equation looks just like the free-particle case:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE'}{h^2}\psi = 0.$$

The solution is $\psi = \psi_0 \exp(ik'x)$, where

$$k'^2 = \frac{8\pi^2 mE'}{h^2} \quad \Rightarrow \quad k = \frac{2\pi}{h}\sqrt{2mE'} = \frac{2\pi}{h}\sqrt{2m(E - U_0)}\ .$$

19. **THINK** Even though $E < U_b$, barrier tunneling can still take place quantum mechanically with finite probability.

EXPRESS If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U_b and width L is given by $T = e^{-2bL}$, where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

If the change ΔU_b in U_b is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU_b}\Delta U_b = -2LT\frac{db}{dU_b}\Delta U_b.$$

Now,

$$\frac{db}{dU_b} = \frac{1}{2\sqrt{U_b - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U_b - E)}\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}} = \frac{b}{2(U_b - E)}.$$

Thus,

$$\Delta T = -LTb\frac{\Delta U_b}{U_b - E}.$$

ANALYZE (a) With

$$b = \sqrt{\frac{8\pi^2\left(9.11\times10^{-31}\text{ kg}\right)\left(6.8\text{ eV} - 5.1\text{ eV}\right)\left(1.6022\times10^{-19}\text{ J/eV}\right)}{\left(6.6261\times10^{-34}\text{ J}\cdot\text{s}\right)^2}} = 6.67\times10^{9}\text{ m}^{-1},$$

we have $bL = (6.67\times10^{9}\text{ m}^{-1})(750\times10^{-12}\text{ m}^{-1}) = 5.0,$ and

$$\frac{\Delta T}{T} = -bL\frac{\Delta U_b}{U_b - E} = -(5.0)\frac{(0.010)(6.8\text{ eV})}{6.8\text{ eV} - 5.1\text{ eV}} = -0.20.$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\Delta L = -2be^{-2bL}\Delta L = -2bT\Delta L$$

and

$$\frac{\Delta T}{T} = -2b\Delta L = -2\left(6.67\times10^{9}\text{ m}^{-1}\right)(0.010)\left(750\times10^{-12}\text{ m}\right) = -0.10.$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\Delta E = -2Le^{-2bL}\frac{db}{dE}\Delta E = -2LT\frac{db}{dE}\Delta E.$$

Now, $db/dE = -db/dU_b = -b/2\left(U_b - E\right)$, so

$$\frac{\Delta T}{T} = bL\frac{\Delta E}{U_b - E} = \left(5.0\right)\frac{\left(0.010\right)\left(5.1\,\text{eV}\right)}{6.8\,\text{eV} - 5.1\,\text{eV}} = 0.15\ .$$

There is a 15% increase in the transmission coefficient.

LEARN Increasing the barrier height or the barrier thickness reduces the probability of transmission, while increasing the kinetic energy of the electron increases the probability.

51. (a) We use Eq. 38-6:

$$V_{\text{stop}} = \frac{hf - \Phi}{e} = \frac{hc/\lambda - \Phi}{e} = \frac{(1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}) - 1.8\,\text{eV}}{e} = 1.3\,\text{V}.$$

(b) The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2} m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v = \sqrt{\frac{2\left(E_{\text{photon}} - \Phi\right)}{m_e}} = \sqrt{\frac{2eV_{\text{stop}}}{m_e}} = c\sqrt{\frac{2eV_{\text{stop}}}{m_e c^2}} = \left(2.998\times10^8\,\text{m/s}\right)\sqrt{\frac{2e(1.3\,\text{V})}{511\times10^3\,\text{eV}}}$$
$$= 6.8\times10^5\,\text{m/s}.$$