23. (a) We denote the upper level as level 1 and the lower one as level 2. From $N_1/N_2 = e^{-(E_2-E_1)/kT}$ we get (using hc = 1240 eV·nm)

$$N_1 = N_2 e^{-(E_1-E_2)/kT} = N_2 e^{-hc/\lambda kT} = \left(4.0\times10^{20}\right)\exp\left[-\frac{1240\,\text{eV}\cdot\text{nm}}{(580\,\text{nm})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}\right]$$
$$= 5.0\times10^{-16} << 1,$$

so practically no electron occupies the upper level.

(b) With $N_1 = 3.0 \times 10^{20}$ atoms emitting photons and $N_2 = 1.0 \times 10^{20}$ atoms absorbing photons, then the net energy output is

$$E = \left(N_1 - N_2\right)E_{\text{photon}} = \left(N_1 - N_2\right)\frac{hc}{\lambda} = \left(2.0\times10^{20}\right)\frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\,\text{m/s}\right)}{580\times10^{-9}\,\text{m}}$$
$$= 68\,\text{J}.$$

26. The energy of the laser pulse is

$$E_p = P\Delta t = (3.25\times10^6 \text{ J/s})(0.500\times10^{-6} \text{ s}) = 1.63 \text{ J} .$$

Since the energy carried by each photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^8 \text{ m/s})}{424\times10^{-9}\text{ m}} = 4.69\times10^{-19}\text{ J},$$

the number of photons emitted in each pulse is

$$N = \frac{E_p}{E} = \frac{1.63\text{J}}{4.69\times10^{-19}\text{ J}} = 3.5\times10^{18} \text{ photons.}$$

With each atom undergoing stimulated emission only once, the number of atoms contributed to the pulse is also 3.5×10^{18}.

36. With hc = 1240 eV·nm = 1240 keV·pm, for the K_α line from iron, the energy difference is

$$\Delta E = \frac{hc}{\lambda} = \frac{1240\,\text{keV}\cdot\text{pm}}{193\,\text{pm}} = 6.42\ \text{keV}.$$

We remark that for the hydrogen atom the corresponding energy difference is

$$\Delta E_{12} = -(13.6\,\text{eV})\left(\frac{1}{2^2} - \frac{1}{1^1}\right) = 10\,\text{eV}\ .$$

That this difference is much greater in iron is due to the fact that its atomic nucleus contains 26 protons, exerting a much greater force on the K- and L-shell electrons than that provided by the single proton in hydrogen.

49. THINK Knowing the value of ℓ, the orbital quantum number, allows us to determine the magnitudes of the angular momentum and the magnetic dipole moment.

EXPRESS The magnitude of the orbital angular momentum is

$$L=\sqrt{\ell(\ell+1)}\hbar .$$

Similarly, with $\vec{\mu}_{\text{orb}}=-\dfrac{e}{2m}\vec{L},$ the magnitude of $\vec{\mu}_{\text{orb}}$ is

$$\mu_{\text{orb}}=\frac{e\hbar}{2m}\sqrt{\ell(\ell+1)}=\mu_{\text{B}},$$

where $\mu_{\text{B}}=e\hbar/2m$ is the Bohr magneton.

ANALYZE (a) For $\ell=3,$ we have

$$L=\sqrt{\ell(\ell+1)}\hbar=\sqrt{3(3+1)}\hbar=\sqrt{12}\hbar .$$

So the multiple is $\sqrt{12}\approx 3.46.$

(b) The magnitude of the orbital dipole moment is

$$\mu_{\text{orb}}=\sqrt{\ell(\ell+1)}\mu_B=\sqrt{12}\mu_B .$$

So the multiple is $\sqrt{12}\approx 3.46.$

(c) The largest possible value of m_ℓ is $m_\ell=\ell=3$.

(d) We use $L_z=m_\ell\hbar$ to calculate the z component of the orbital angular momentum. The multiple is $m_\ell=3$.

(e) We use $\mu_z=-m_\ell\mu_B$ to calculate the z component of the orbital magnetic dipole moment. The multiple is $-m_\ell=-3$.

(f) We use $\cos\theta=m_\ell\big/\sqrt{\ell(\ell+1)}$ to calculate the angle between the orbital angular momentum vector and the z axis. For $\ell=3$ and $m_\ell=3$, we have $\cos\theta=3/\sqrt{12}=\sqrt{3}/2$, or $\theta=30.0°$.

(g) For $\ell=3$ and $m_\ell=2$, we have $\cos\theta=2/\sqrt{12}=1/\sqrt{3}$, or $\theta=54.7°$.

(h) For $\ell=3$ and $m_\ell=-3$, $\cos\theta=-3/\sqrt{12}=-\sqrt{3}/2$, or $\theta=150°$.

LEARN Neither $\vec{L}$ nor $\vec{\mu}_{\text{orb}}$ can be measured in any way. We can, however, measure their z components.

53. **THINK** With eight electrons, the ground-state energy of the system is the sum of the energies of the individual electrons in the system's ground-state configuration.

EXPRESS In terms of the quantum numbers n_x, n_y, and n_z, the single-particle energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right).$$

The lowest single-particle level corresponds to $n_x = 1$, $n_y = 1$, and $n_z = 1$ and is $E_{1,1,1} = 3(h^2/8mL^2)$. There are two electrons with this energy, one with spin up and one with spin down. The next lowest single-particle level is three-fold degenerate in the three integer quantum numbers. The energy is

$$E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2).$$

Each of these states can be occupied by a spin up and a spin down electron, so six electrons in all can occupy the states. This completes the assignment of the eight electrons to single-particle states.

ANALYZE The ground state energy of the system is

$$E_{gr} = (2)(3)(h^2/8mL^2) + (6)(6)(h^2/8mL^2) = 42(h^2/8mL^2).$$

Thus, the multiple of $h^2/8mL^2$ is 42.

LEARN We summarize the ground-state configuration and the energies (in multiples of $h^2/8mL^2$) in the chart below:

n_x	n_y	n_z	m_s	energy
1	1	1	–1/2, + 1/2	3 + 3
1	1	2	–1/2, + 1/2	6 + 6
1	2	1	–1/2, + 1/2	6 + 6
2	1	1	–1/2, + 1/2	6 + 6
			total	42