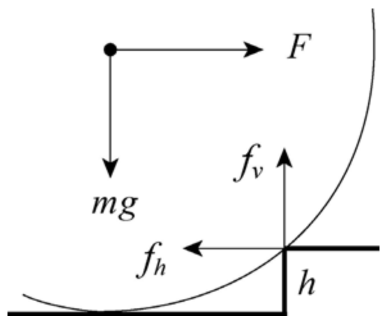
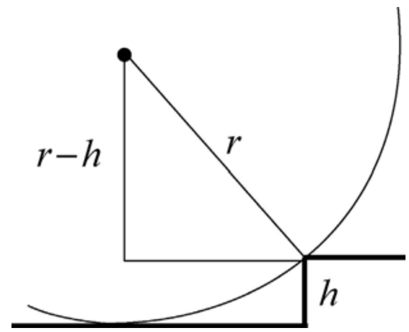
25. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.





We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step.

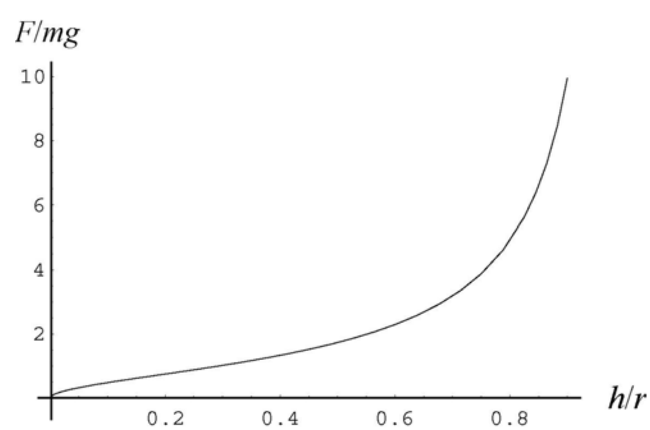
The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus,

$$F(r-h) - mg\sqrt{2rh - h^2} = 0 .$$

The solution for F is

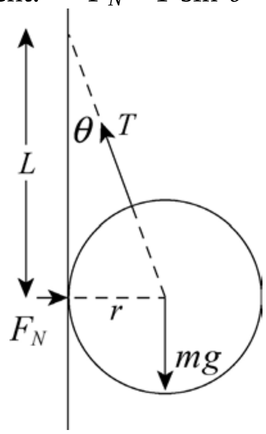
$$F = \frac{\sqrt{2rh - h^2}}{r-h} mg = \frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})}(0.800\text{ kg})(9.80\text{ m/s}^2)$$
$$= 13.6\text{ N}.$$

Note: The applied force here is about 1.73 times the weight of the wheel. If the height is increased, the force that must be applied also goes up. Next we plot F/mg as a function of the ratio h/r. The required force increases rapidly as $h/r \to 1$.



3. Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives

$$\text{vertical component: } \quad T\cos\theta - mg = 0$$
$$\text{horizontal component: } \quad F_N - T\sin\theta = 0.$$



(a) We solve the first equation for the tension and obtain $T = mg/\cos\theta$. We then substitute $\cos\theta = L/\sqrt{L^2+r^2}$:

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)\sqrt{(0.080\text{ m})^2+(0.042\text{ m})^2}}{0.080\text{ m}} = 9.4\text{ N}.$$

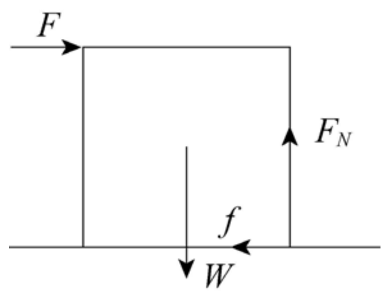
(b) We solve the second equation for the normal force and obtain $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we have

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L}$$
$$= \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)(0.042\text{ m})}{(0.080\text{ m})} = 4.4\text{ N}.$$

35. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram that follows. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

$$F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\,\text{N}.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium

$$F_N = W = 890\text{ N},\ \ f = F = 445\text{ N},$$

so

$$\mu_s = \frac{f}{F_N} = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

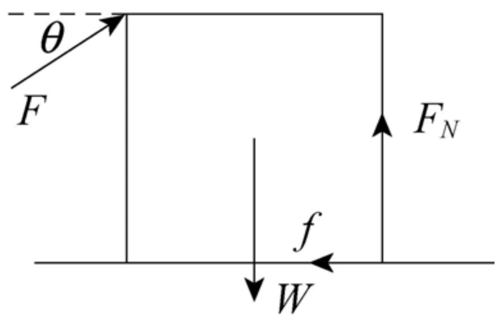
$$FL\cos\theta + FL\sin\theta - WL/2 = 0,$$

with the solution

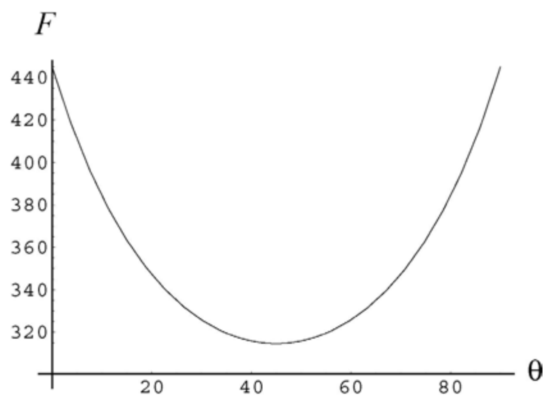
$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

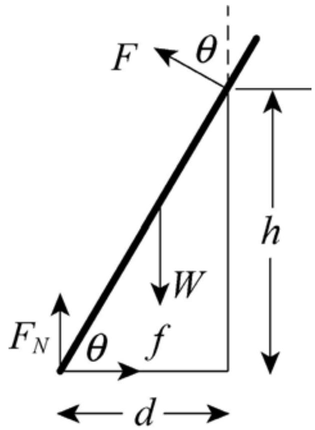
$$F = \frac{W}{2(\cos 45° + \sin 45°)} = \frac{890\,\text{N}}{2(\cos 45° + \sin 45°)} = 315\,\text{N}.$$



Note: The applied force as a function of θ is plotted below. From the figure, we readily see that $\theta = 0°$ corresponds to a maximum and $\theta = 45°$ to a minimum.



37. The free-body diagram below shows the forces acting on the plank. Since the roller is frictionless, the force it exerts is normal to the plank and makes the angle θ with the vertical.



Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.

The equations of equilibrium are:

horizontal force components: $F\sin\theta - f = 0$

vertical force components: $F\cos\theta - W + F_N = 0$

torques: $F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70^\circ$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70^\circ$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have used $d = h/\tan\theta$ and multiplied both numerator and denominator by tan θ. We use the trigonometric identity $1 + \tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

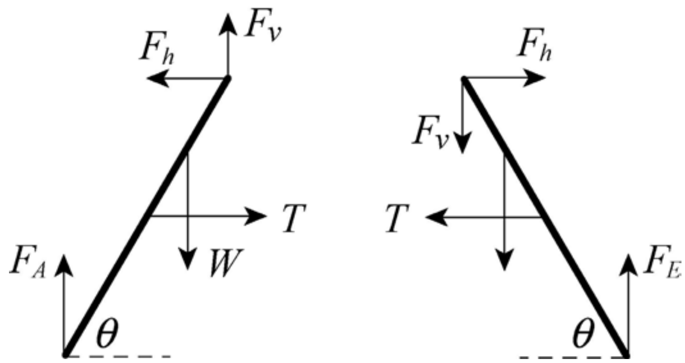
Substituting these expressions for f and F_N into $\mu_s = f/F_N$ leads to

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70°$, $L = 6.10$ m and $h = 3.05$ m gives

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70°\cos 70°}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70°\cos^2 70°} = 0.34.$$

39. The diagrams show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction. Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $F_v + F_A - W = 0$. The horizontal components must sum to zero: $T - F_h = 0$.



The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then

$$F_A L \cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta = 0.$$

Here we recognize that the man is a distance d from the bottom of the ladder (or $L - d$ from the top), and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L \cos\theta - T(L/2)\sin\theta = 0$. There are 5 different equations:

$$\begin{aligned} F_v + F_A - W &= 0, \\ T - F_h &= 0 \\ F_A L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E - F_v &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$T - F_h = 0$$
$$WL\cos\theta - F_E L\cos\theta - W(L-d)\cos\theta - T(L/2)\sin\theta = 0$$
$$F_E L\cos\theta - T(L/2)\sin\theta = 0.$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{Wd}{L\tan\theta}.$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of

$\sqrt{(1.22\text{ m})^2 - (0.381\text{ m})^2} = 1.16\text{ m}$. This means

$\tan\theta = (1.16\text{m})/(0.381\text{m}) = 3.04.$

Thus,

$$T = \frac{(854\text{ N})(1.80\text{ m})}{(2.44\text{ m})(3.04)} = 207\text{ N}.$$

(b) We now solve for F_A. We substitute $F_v = F_E = (T/2)\tan\theta = Wd/2L$ into the equation $F_v + F_A - W = 0$ and solve for F_A. The solution is

$$F_A = W - F_v = W\left(1 - \frac{d}{2L}\right) = (854\text{ N})\left(1 - \frac{1.80\text{ m}}{2(2.44\text{ m})}\right) = 539\text{ N}.$$

(c) Similarly, $F_E = W\dfrac{d}{2L} = (854\text{ N})\dfrac{1.80\text{ m}}{2(2.44\text{ m})} = 315\text{ N}$.