

39. A Carnot refrigerator working between a hot reservoir at temperature  $T_H$  and a cold reservoir at temperature  $T_L$  has a coefficient of performance  $K$  that is given by

$$K = \frac{T_L}{T_H - T_L}.$$

For the refrigerator of this problem,  $T_H = 96^\circ \text{ F} = 309 \text{ K}$  and  $T_L = 70^\circ \text{ F} = 294 \text{ K}$ , so

$$K = (294 \text{ K}) / (309 \text{ K} - 294 \text{ K}) = 19.6.$$

The coefficient of performance is the energy  $Q_L$  drawn from the cold reservoir as heat divided by the work done:  $K = |Q_L|/|W|$ . Thus,

$$|Q_L| = K|W| = (19.6)(1.0 \text{ J}) = 20 \text{ J}.$$

18. (a) It is possible to motivate, starting from Eq. 20-3, the notion that heat may be found from the integral (or “area under the curve”) of a curve in a  $TS$  diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a “straight-line” path in the  $TS$  diagram:

$$Q_{\text{straight}} = \left( \frac{T_i + T_f}{2} \right) \Delta S$$

which could, in fact, be *directly* motivated from Eq. 20-3 (but it is important to bear in mind that this is rigorously true only for a process that forms a straight line in a graph that plots  $T$  versus  $S$ ). This leads to

$$Q = (300 \text{ K}) (15 \text{ J/K}) = 4.5 \times 10^3 \text{ J}$$

for the energy absorbed as heat by the gas.

(b) Using Table 19-3 and Eq. 19-45, we find

$$\Delta E_{\text{int}} = n \left( \frac{3}{2} R \right) \Delta T = (2.0 \text{ mol}) (8.31 \text{ J/mol} \cdot \text{K}) (200 \text{ K} - 400 \text{ K}) = -5.0 \times 10^3 \text{ J}.$$

(c) By the first law of thermodynamics,

$$W = Q - \Delta E_{\text{int}} = 4.5 \text{ kJ} - (-5.0 \text{ kJ}) = 9.5 \text{ kJ}.$$

46. (a) We denote the configuration with  $n$  heads out of  $N$  trials as  $(n; N)$ . We use Eq. 20-20:

$$W(25; 50) = \frac{50!}{(25!)(50-25)!} = 1.26 \times 10^{14}.$$

(b) There are 2 possible choices for each molecule: it can either be in side 1 or in side 2 of the box. If there are a total of  $N$  independent molecules, the total number of available states of the  $N$ -particle system is

$$N_{\text{total}} = 2 \times 2 \times 2 \times \cdots \times 2 = 2^N.$$

With  $N = 50$ , we obtain  $N_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$ .

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25; 50) = \frac{W(25; 50)}{2^{50}} = \frac{1.26 \times 10^{14}}{1.13 \times 10^{15}} = 11.1\%.$$

With  $N = 100$ , we obtain

(d)  $W(N/2, N) = N! / [(N/2)!]^2 = 1.01 \times 10^{29}$ ,

(e)  $N_{\text{total}} = 2^N = 1.27 \times 10^{30}$ ,

(f) and  $p(N/2; N) = W(N/2, N) / N_{\text{total}} = 8.0\%$ .

Similarly, for  $N = 200$ , we obtain

(g)  $W(N/2, N) = 9.25 \times 10^{58}$ ,

(h)  $N_{\text{total}} = 1.61 \times 10^{60}$ ,

(i) and  $p(N/2; N) = 5.7\%$ .

(j) As  $N$  increases, the number of available microscopic states increase as  $2^N$ , so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration. Thus, the time spent in there decreases with an increase in  $N$ .

33. (a) Energy is added as heat during the portion of the process from  $a$  to  $b$ . This portion occurs at constant volume ( $V_b$ ), so  $Q_{\text{in}} = nC_V \Delta T$ . The gas is a monatomic ideal gas, so  $C_V = 3R/2$  and the ideal gas law gives

$$\Delta T = (1/nR)(p_b V_b - p_a V_a) = (1/nR)(p_b - p_a) V_b.$$

Thus,  $Q_{\text{in}} = \frac{3}{2}(p_b - p_a)V_b$ .  $V_b$  and  $p_b$  are given. We need to find  $p_a$ . Now  $p_a$  is the same as  $p_c$ , and points  $c$  and  $b$  are connected by an adiabatic process. Thus,  $p_c V_c^\gamma = p_b V_b^\gamma$  and

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3} (1.013 \times 10^6 \text{ Pa}) = 3.167 \times 10^4 \text{ Pa}.$$

The energy added as heat is

$$Q_{\text{in}} = \frac{3}{2}(1.013 \times 10^6 \text{ Pa} - 3.167 \times 10^4 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3) = 1.47 \times 10^3 \text{ J}.$$

(b) Energy leaves the gas as heat during the portion of the process from  $c$  to  $a$ . This is a constant pressure process, so

$$\begin{aligned} Q_{\text{out}} &= nC_p \Delta T = \frac{5}{2}(p_a V_a - p_c V_c) = \frac{5}{2} p_a (V_a - V_c) \\ &= \frac{5}{2}(3.167 \times 10^4 \text{ Pa})(-7.00)(1.00 \times 10^{-3} \text{ m}^3) = -5.54 \times 10^2 \text{ J}, \end{aligned}$$

or  $|Q_{\text{out}}| = 5.54 \times 10^2 \text{ J}$ . The substitutions  $V_a - V_c = V_a - 8.00 V_a = -7.00 V_a$  and  $C_p = \frac{5}{2} R$  were made.

(c) For a complete cycle, the change in the internal energy is zero and

$$W = Q = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}.$$

(d) The efficiency is

$$\varepsilon = W/Q_{\text{in}} = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624 = 62.4\%.$$

11. (a) The energy that leaves the aluminum as heat has magnitude  $Q = m_a c_a (T_{ai} - T_f)$ , where  $m_a$  is the mass of the aluminum,  $c_a$  is the specific heat of aluminum,  $T_{ai}$  is the initial temperature of the aluminum, and  $T_f$  is the final temperature of the aluminum–water system. The energy that enters the water as heat has magnitude  $Q = m_w c_w (T_f - T_{wi})$ , where  $m_w$  is the mass of the water,  $c_w$  is the specific heat of water, and  $T_{wi}$  is the initial temperature of the water. The two energies are the same in magnitude, since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \Rightarrow T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}.$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K})(100^\circ\text{C}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})(20^\circ\text{C})}{(0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) + (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K})} \\ &= 57.0^\circ\text{C} = 330 \text{ K}. \end{aligned}$$

(b) Now temperatures must be given in Kelvins:  $T_{ai} = 393 \text{ K}$ ,  $T_{wi} = 293 \text{ K}$ , and  $T_f = 330 \text{ K}$ . For the aluminum,  $dQ = m_a c_a dT$ , and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} = (0.200 \text{ kg})(900 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{330 \text{ K}}{373 \text{ K}} \right) \\ &= -22.1 \text{ J/K}. \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} = (0.0500 \text{ kg})(4190 \text{ J/kg} \cdot \text{K}) \ln \left( \frac{330 \text{ K}}{293 \text{ K}} \right) \\ &= +24.9 \text{ J/K}. \end{aligned}$$

(d) The change in the total entropy of the aluminum–water system is

$$\Delta S = \Delta S_a + \Delta S_w = -22.1 \text{ J/K} + 24.9 \text{ J/K} = +2.8 \text{ J/K}.$$

27. (a) The efficiency is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(235 - 115) \text{ K}}{(235 + 273) \text{ K}} = 0.236 = 23.6\% .$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by  $\varepsilon = |W|/|Q_H|$ , the work done is given by

$$|W| = \varepsilon |Q_H| = 0.236(6.30 \times 10^4 \text{ J}) = 1.49 \times 10^4 \text{ J} .$$