7. In this problem we have two forces acting on a box to produce a given acceleration. We apply Newton's second law to solve for the unknown second force. We denote the two forces as $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so the second force is $\vec{F}_2 = m\vec{a} - \vec{F}_1$. Note that since the acceleration is in the third quadrant, we expect $\vec{F}_2$ to be in the third quadrant as well.

(a) In unit vector notation $\vec{F}_1 = (20.0\text{ N})\hat{\text{i}}$ and

$$\vec{a} = -\left(12.0\sin 30.0°\,\text{m/s}^2\right)\hat{\text{i}} - \left(12.0\cos 30.0°\,\text{m/s}^2\right)\hat{\text{j}} = -\left(6.00\text{ m/s}^2\right)\hat{\text{i}} - \left(10.4\text{m/s}^2\right)\hat{\text{j}}.$$

Therefore, we find the second force to be

$$\begin{aligned}\vec{F}_2 &= m\vec{a} - \vec{F}_1\\ &= (2.00\,\text{kg})\left(-6.00\,\text{m/s}^2\right)\hat{\text{i}} + (2.00\text{ kg})\left(-10.4\text{ m/s}^2\right)\hat{\text{j}} - (20.0\text{ N})\hat{\text{i}}\\ &= (-32.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}.\end{aligned}$$

(b) The magnitude of $\vec{F}_2$ is $|\vec{F}_2| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32.0\text{ N})^2 + (-20.8\text{ N})^2} = 38.2\text{ N}.$

(c) The angle that $\vec{F}_2$ makes with the positive x-axis is found from

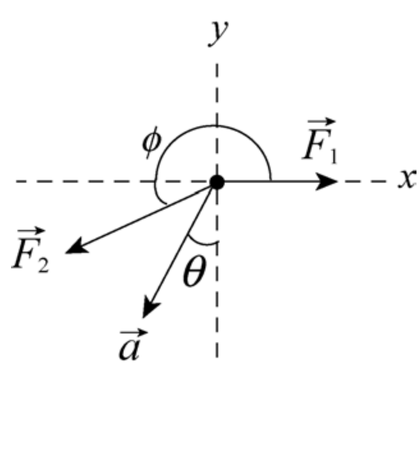
$$\tan\phi = \left(\frac{F_{2y}}{F_{2x}}\right) = \frac{-20.8\text{ N}}{-32.0\text{ N}} = 0.656$$

Consequently, the angle is either 33.0° or 33.0° + 180° = 213°. Since both the x and y components are negative, the correct result is ϕ = 213° from the +x-axis. An alternative answer is $213° - 360° = -147°$.

The result is depicted to the right. The calculation confirms our expectation that $\vec{F}_2$ lies in the third quadrant (same as $\vec{a}$). The net force is

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_1 + \vec{F}_2 = (20.0\text{ N})\hat{\text{i}} + \left[(-32.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}\right]\\ &= (-12.0\text{ N})\hat{\text{i}} - (20.8\text{ N})\hat{\text{j}}\end{aligned}$$

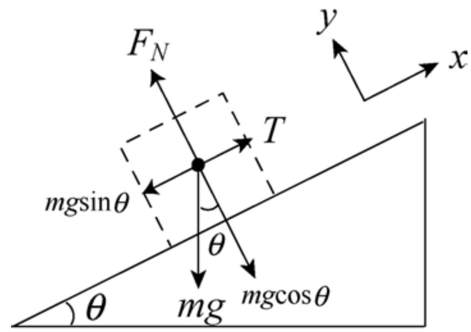
which points in the same direction as $\vec{a}$.



17. The free-body diagram of the problem is shown to the right. Since the acceleration of the block is zero, the components of the Newton's second law equation yield

$$T - mg\sin\theta = 0$$
$$F_N - mg\cos\theta = 0,$$

where T is the tension in the cord, and F_N is the normal force on the block.



(a) Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\ \text{kg})(9.8\ \text{m/s}^2)\ \sin 30° = 42\ \text{N}\ .$$

(b) We solve the second equation in part (a) for the normal force F_N:

$$F_N = mg\cos\theta = (8.5\ \text{kg})(9.8\ \text{m/s}^2)\ \cos\ 30° = 72\ \text{N}\ .$$

(c) When the cord is cut, it no longer exerts a force on the block and the block accelerates. The x-component equation of Newton's second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

$$a = -g\sin\theta = -(9.8\ \text{m/s}^2)\sin 30° = -4.9\ \text{m/s}^2.$$

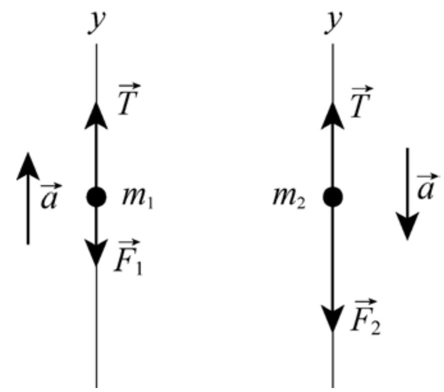
The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

Note: The normal force F_N on the block must be equal to $mg\cos\theta$ so that the block is in contact with the surface of the incline at all time. When the cord is cut, the block has an acceleration $a = -g\sin\theta$, which in the limit $\theta \to 90°$ becomes $-g$.

51. The free-body diagrams for m_1 and m_2 are shown in the figures below. The only forces on the blocks are the upward tension $\vec{T}$ and the downward gravitational forces $\vec{F}_1 = m_1 g$ and $\vec{F}_2 = m_2 g$. Applying Newton's second law, we obtain:

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$



which can be solved to yield

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Substituting the result back, we have

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g$$

(a) With $m_1 = 1.3\ \text{kg}$ and $m_2 = 2.8\ \text{kg}$, the acceleration becomes

$$a = \left(\frac{2.80\ \text{kg} - 1.30\ \text{kg}}{2.80\ \text{kg} + 1.30\ \text{kg}}\right)(9.80\ \text{m/s}^2) = 3.59\ \text{m/s}^2 \approx 3.6\ \text{m/s}^2.$$

(b) Similarly, the tension in the cord is

$$T = \frac{2(1.30\ \text{kg})(2.80\ \text{kg})}{1.30\ \text{kg} + 2.80\ \text{kg}}(9.80\ \text{m/s}^2) = 17.4\ \text{N} \approx 17\ \text{N}.$$

56. Both situations involve the same applied force and the same total mass, so the accelerations must be the same in both figures.

(a) The (direct) force causing *B* to have this acceleration in the first figure is twice as big as the (direct) force causing *A* to have that acceleration. Therefore, *B* has the twice the mass of *A*. Since their total is given as 12.0 kg then *B* has a mass of m_B = 8.00 kg and *A* has mass m_A = 4.00 kg. Considering the first figure, (20.0 N)/(8.00 kg) = 2.50 m/s^2. Of course, the same result comes from considering the second figure ((10.0 N)/(4.00 kg) = 2.50 m/s^2).

(b) F_a = (12.0 kg)(2.50 m/s^2) = 30.0 N

57. The free-body diagram for each block is shown below. *T* is the tension in the cord and θ = 30° is the angle of the incline. For block 1, we take the +*x* direction to be up the incline and the +*y* direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the +*y* direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol *a*, without ambiguity. Applying Newton's second law to the *x* and *y* axes for block 1 and to the *y* axis of block 2, we obtain

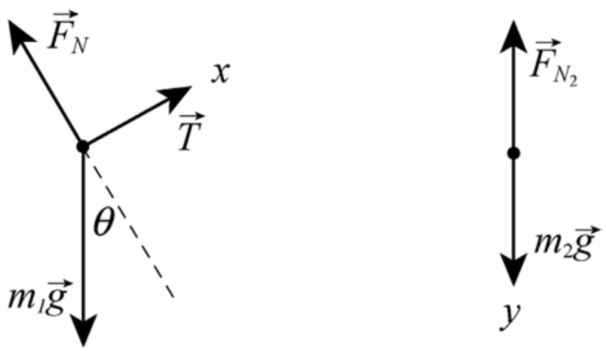
$$\begin{aligned} T - m_1 g \sin\theta &= m_1 a \\ F_N - m_1 g \cos\theta &= 0 \\ m_2 g - T &= m_2 a \end{aligned}$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of *a* and *T*. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).

57. The free-body diagram for each block is shown below. T is the tension in the cord and θ = 30° is the angle of the incline. For block 1, we take the $+x$ direction to be up the incline and the $+y$ direction to be in the direction of the normal force $\vec{F}_N$ that the plane exerts on the block. For block 2, we take the $+y$ direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol a, without ambiguity. Applying Newton's second law to the x and y axes for block 1 and to the y axis of block 2, we obtain

$$\begin{aligned} T - m_1 g \sin\theta &= m_1 a \\ F_N - m_1 g \cos\theta &= 0 \\ m_2 g - T &= m_2 a \end{aligned}$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of a and T. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).



(a) We add the first and third equations above:

$$m_2 g - m_1 g \sin\theta = m_1 a + m_2 a.$$

Consequently, we find

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} = \frac{[2.30\text{ kg} - (3.70\text{ kg})\sin 30.0°](9.80\text{ m/s}^2)}{3.70\text{ kg} + 2.30\text{ kg}} = 0.735\text{m/s}^2.$$

(b) The result for a is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1 a + m_1 g \sin\theta = (3.70\text{ kg})(0.735\text{ m/s}^2) + (3.70\text{ kg})(9.80\text{ m/s}^2)\sin 30.0° = 20.8\text{N}.$$

59. We take $+y$ to be up for both the monkey and the package. The force the monkey pulls downward on the rope has magnitude F. According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to

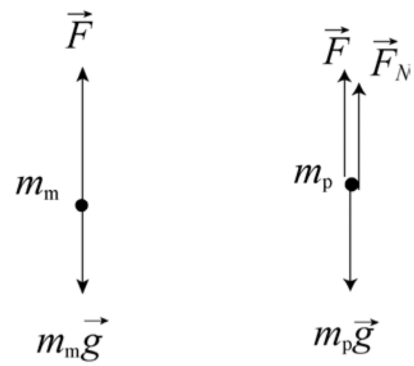
$$F - m_m g = m_m a_m,$$

where m_m is the mass of the monkey and a_m is its acceleration. Since the rope is massless $F = T$ is the tension in the rope.

The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is

$$F + F_N - m_p g = m_p a_p,$$

where m_p is the mass of the package, a_p is its acceleration, and F_N is the normal force exerted by the ground on it. The free-body diagrams for the monkey and the package are shown to the right (not to scale).



Now, if F is the minimum force required to lift the package, then $F_N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$.

(a) Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)g}{m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{10\text{ kg}} = 4.9\text{ m/s}^2.$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a'_p$ for the package and $F - m_m g = m_m a'_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a'_m = -a'_p$. Solving the first equation for F

$$F = m_p\left(g + a'_p\right) = m_p\left(g - a'_m\right)$$

and substituting this result into the second equation:

$$m_p(g - a'_m) - m_m g = m_m a'_m,$$

we solve for a'_m:

$$a'_m = \frac{(m_p - m_m)g}{m_p + m_m} = \frac{(15\text{ kg} - 10\text{ kg})(9.8\text{ m/s}^2)}{15\text{ kg} + 10\text{ kg}} = 2.0\text{ m/s}^2.$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, the tension in the rope is

$$F = m_p\left(g - a'_m\right) = (15\text{ kg})\left(9.8\text{ m/s}^2 - 2.0\text{ m/s}^2\right) = 120\text{N}.$$