1. The potential energy stored by the spring is given by $U = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\,\text{J})}{(0.075\,\text{m})^2} = 8.9 \times 10^3\ \text{N/m}.$$

5. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude *mg*, where *m* is the mass of the flake, so this reduces to *W* = *mgh*, where *h* is the height from which the flake falls. This is equal to the radius *r* of the bowl. Thus

$$W = mgr = (2.00 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)(22.0 \times 10^{-2}\,\text{m}) = 4.31 \times 10^{-3}\,\text{J}.$$

(b) The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W = -4.31 \times 10^{-3}$ J.

(c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

(d) If $U = 0$ at the top, then $U = -\,4.31 \times 10^{-3}$ J at the bottom.

(e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

11. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$.

Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where r = 0.220 m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f. Energy conservation leads to

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\,(1-\cos\theta)\ .$$

The speed is $v = \sqrt{2gr} = \sqrt{2(9.8\ \text{m/s}^2)(0.220\ \text{m})} = 2.08\ \text{m/s}$.

(b) Since the expression for speed does not contain the mass of the flake, the speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

24. We denote m as the mass of the block, h = 0.40 m as the height from which it dropped (measured from the relaxed position of the spring), and x as the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f$$
$$0 = -mg(h + x) + \frac{1}{2}kx^2$$

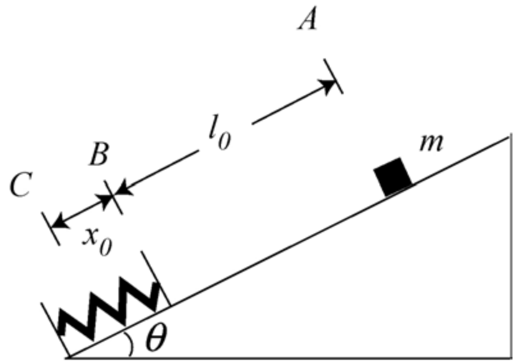
which is a second degree equation in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} .$$

Now mg = 19.6 N, h = 0.40 m, and $k = 1960\ \mathrm{N/m}$, and we choose the positive root so that $x > 0$.

$$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10\ \mathrm{m} .$$

29. We refer to its starting point as *A*, the point where it first comes into contact with the spring as *B*, and the point where the spring is compressed $x_0 = 0.055$ m as *C*, as shown in the figure below. Point *C* is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed.



Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270 \text{ N}}{0.02 \text{ m}} = 1.35 \times 10^4 \text{ N/m} .$$

The distance between points *A* and *B* is l_0 and we note that the total sliding distance $l_0 + x_0$ is related to the initial height h_A of the block (measured relative to *C*) by

$$\sin\theta = \frac{h_A}{l_0 + x_0}$$

where the incline angle θ is 30°.

(a) Mechanical energy conservation leads to

$$K_A + U_A = K_C + U_C \quad \Rightarrow \quad 0 + mgh_A = \frac{1}{2}kx_0^2$$

which yields

$$h_A = \frac{kx_0^2}{2mg} = \frac{(1.35 \times 10^4 \text{ N/m})(0.055 \text{ m})^2}{2(12 \text{ kg})(9.8 \text{ m/s}^2)} = 0.174 \text{ m} .$$

Therefore, the total distance traveled by the block before coming to a stop is

$$l_0 + x_0 = \frac{h_A}{\sin 30°} = \frac{0.174 \text{ m}}{\sin 30°} = 0.347 \text{ m} \approx 0.35 \text{ m} .$$

(b) From this result, we find $l_0 = x_0 = 0.347 \text{ m} - 0.055 \text{ m} = 0.292 \text{ m}$, which means that the block has descended a vertical distance

$$|\Delta y| = h_A - h_B = l_0 \sin\theta = (0.292\ \text{m})\sin 30° = 0.146\ \text{m}$$

in sliding from point *A* to point *B*. Thus, using Eq. 8-18, we have

$$0 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B \quad \Rightarrow \quad \frac{1}{2}mv_B^2 = mg\,|\Delta y|$$

which yields $v_B = \sqrt{2g\,|\Delta y|} = \sqrt{2(9.8\ \text{m/s}^2)(0.146\ \text{m})} = 1.69\ \text{m/s} \approx 1.7\ \text{m/s}$.

Note: Energy is conserved in the process. The total energy of the block at position *B* is

$$E_B = \frac{1}{2}mv_B^2 + mgh_B = \frac{1}{2}(12\ \text{kg})(1.69\ \text{m/s})^2 + (12\ \text{kg})(9.8\ \text{m/s}^2)(0.028\ \text{m}) = 20.4\ \text{J},$$

which is equal to the elastic potential energy in the spring:

$$\frac{1}{2}kx_0^2 = \frac{1}{2}(1.35\times10^4\ \text{N/m})(0.055\ \text{m})^2 = 20.4\ \text{J}.$$

34. Let $\vec{F}_N$ be the normal force of the ice on him and *m* is his mass. The net inward force is $mg\cos\theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where *v* is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is

$$U = -mgR(1 - \cos\theta).$$

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives

$$0 = \tfrac{1}{2}mv^2 - mgR(1 - \cos\theta),$$

or $v^2 = 2gR(1 - \cos\theta)$. We substitute this expression into the equation developed from the second law to obtain $g\cos\theta = 2g(1 - \cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R\cos\theta = \frac{2}{3}R = \frac{2}{3}(13.8\ \text{m}) = 9.20\ \text{m}.$$