Sometimes people seem to continue investing after they realize that the investment turns out to be a failure. Usually, the investor can minimize the loss by stopping the investment as soon as they realize that it would be unprofitable.

This phenomenon is explained in this paper with the assumption that the investor concerns about his reputation and other future opportunities. In this model we can also see how the investor's concern on his reputation may change his decision making on the current investment.

Key Words: Failure, Reputation

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I. Introduction

According to economic theory a decision maker should ignore the sunk cost once the cost is made. Usually, people make an initial investment expecting a positive profit. Most of the projects need continuous investment over time. Sometimes, however, the expectation on the profit changes and the on-going investment turns out to be a failure which will end up with a negative profit. At this point the previous investment can be interpreted as a sunk cost, which should not be considered in the investment decision making according to economics textbooks. Non-economists share the same thinking when they say that bygones should be bygones.

In the real world, however, we often find out that people continue to invest on what turns out to be a failed project. It seems as if people’s behavior changes because of the sunk cost. These lingering effects of the sunk cost have appeared in several experiments such as Arkes and Blumer (1985), Frisch (1993), and Keasey and Moon (2000). Also, Heath, Huddart, and Lang (1999) showed that small investors in financial market often sell their winners and hang on to their losers.

Carmichael and MacLeod (2003) had a theoretical model to explain why people would care about the sunk cost in a bargaining game. However, there have not been much of theoretical studies to explain the lingering effect of sunk cost, even though it seems that people care about the sunk cost in the real world.

In this paper we will consider a situation where the current investment decisions may influence the next investment opportunities. Because of the future reputation, a person may continue investing even though it turns out to be unprofitable after the initial investment was made. Also, this future reputation may influence the decision whether a person would initiate the investment and sunk the cost in the first place or not. We will see that under some conditions the future reputation encourages the investors to make the initial investment, while under some other conditions the future reputation discourages the investors to make the initial investment.
Let’s consider it more carefully. When a person initiates an investment with a large initial sunk cost, he must have expected it as a profitable investment from which he can recover more than the sunk cost as a return. This expectation may turn out to be right or wrong. However, if the investor stops a necessary continuous investment after the large sunk cost, it would give a signal to others that his expectation turned out to be incorrect. Then this turned-out-to-be incorrect expectation can be interpreted as a lower ability of that investor because the failure might have been caused by a bad decision or a lack of resourcefulness of the investor. If the investor wants to have further investment opportunities in the future, this kind of bad reputation would be undesirable. As a result, once an investor has invested in a sunk cost, he may continue to invest even though it became obvious that the current investment is not profitable, because the investor wants to maintain a good reputation for the future investment opportunities.

We may understand the previously mentioned empirical results where people continue to invest on obviously failed project can be understood if people care about reputation and future opportunities as well as the current investment profits.

In addition to showing these reputation effects after the initial cost was sunk, we will analyze the effects of the reputation on the decision to make the initial investment or not. In some cases this kind of reputation effect discourages investors from making the initial investment.

Ⅱ. Model

We will consider a two period model. An investor will get a return of \(\pi_s (\pi_s > 0)\) from the investment. The return can be realized only after the investor invests a sunk cost \(S\) in the first period \((t = 1)\) and invests an additional cost \(I\) in the second period \((t = 2)\). Unless the investor invests both \(S\) and \(I\), the return will be zero. If both \(S\) and \(I\) are invested, the profit would be \(\pi = \pi_s - S - I\).
The size of the sunk cost, \( S \), is fixed and known to the investor from the beginning, while the size of the additional cost, \( I \), would be known to the investor only in \( t = 2 \). We will assume that the investor will get a signal on the size of the additional cost, \( I \), before he decides to invest the sunk cost or not in \( t = 1 \). We will denote this signal that the investor receives on \( I \) in \( t = 1 \) as \( \widehat{I} \). In \( t = 2 \) the real additional cost, \( I \), may turn out to be same as the signal, that is \( I = \widehat{I} \), or it may turn out to be higher than the signal \( I > \widehat{I} \). It will be assumed that the investment can be profitable if the signal turns out to be correct, \( \pi = \pi_s - S - I > 0 \), \( (I = \widehat{I}) \) depending on the size of \( \widehat{I} \). However, if the signal turns out to be incorrect and the additional cost is higher than expected, it would always be \( \pi = \pi_s - S - I < 0 \), \( (I > \widehat{I}) \) regardless of the size of \( \widehat{I} \). The investor will get to know the real \( I \) before he makes the decision on the additional investment in \( t = 2 \).

There is only one investor and the investor may be a high type with an ex ante probability \( \gamma \) and a low type with an ex ante probability \( (1 - \gamma) \). The type is private information for the investor himself, while other people only know the ex-ante probability, \( \gamma \), but do not know the investor’s exact type.

As we have already assumed, the investor receives a signal, \( \widehat{I} \), at the beginning of \( t = 1 \), and the investor’s type is related to the quality of this signal. The probability that the additional investment will really turn out to be \( \widehat{I} \) in \( t = 2 \) is \( \alpha_H \) for a high type investor and it is \( \alpha_L \) for a low type investor. These \( \alpha_H \) and \( \alpha_L \) are known to everybody and \( \alpha_H > \alpha_L \). Automatically the additional investment may turn out to be different from \( \widehat{I} \) with probability \( (1 - \alpha_H) \) for the high type investor and \( (1 - \alpha_L) \) for the low type investor.

When the additional investment turns out to be different from the signal of the high type, \( \widehat{I}_H \), it will be \( I_H = \widehat{I}_H + k \ (k > 0) \). On the other hand, when the additional investment turns out to be different from the signal of
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a low type investor, \( \hat{I}_L \), it would become impossible to proceed with the investment for the low type investor and the investment has to stop. We can think about cases where the size of the additional investment is prohibitively big so that a low type investor will make a huge loss and go bankrupt if continues to invest. Alternatively we can interpret this assumption as the low ability of the low type investor when a crisis occurs. A high type investor may conceal the reality that the expectation was wrong and the current investment will certainly be unprofitable. On the other hand, a low type investor may have the ability to conceal the situation from outsiders. If this is true, outsiders may get to know the bad investment result of the low type and the reputation would be ruined whether the low type invests in \( t = 2 \) or not. As a result, the low type will never invest in \( t = 2 \) in this case.

In conclusion there are two differences between the high type and the low type. One difference is that it is more likely that the high type’s signal is correct. The other difference is that even though a high type can still make additional investment after the signal turns out to be incorrect, a low type cannot continue to invest in \( t = 2 \) if the signal turns out to be incorrect. Therefore, the high type is better than the low type in predicting the future cost as well as in reducing the loss when the investment turns out to be a failure.

At the beginning of this paper we have mentioned that the investor would be concerned with future investment opportunities as well as the current investment. We will assume that the investor’s reputation from the current investment has a value of \( A(\mu) \), where \( \mu \) is the belief that the investor is a high type and \( A(\cdot) \) is an increasing function of \( \mu \), \( (dA/d\mu > 0) \). We will assume that \( A(0) = 0 \).

It would be also assumed that only the investor himself can observe whether a profit or a loss was made out of the current investment. The only observation that others can make is whether the investor invested the sunk
cost, $S$ or not in $t = 1$ as well as whether the investor invested the additional cost, $I$ or not in $t = 2$.

We will denote the others’ beliefs that the investor is a high type as $\mu_N$ when he does not invest in $S$, as $\mu_S$ when he invests in $S$ but does not invest in $I$, and as $\mu_I$ when he invested both in $S$ and $I$.

We can summarize the situation as follows. At the beginning of $t = 1$ the investor receives a signal, $\hat{I}_T(T = H, L)$. Then the investor should decide whether to invest the sunk cost or not. At this point the investor only knows that the signal $\hat{I}_T$ may turn out to be correct with a probability, $\alpha_T(T = H, L)$. Once the investor invests the sunk cost, the situation moves to the second period, $t = 2$. At the beginning of $t = 2$, the investor will get to know the real size of the additional investment, $I_T(T = H, L)$ as well as whether the reputation matters or not. Based on this new information, the investor will decide whether to invest the additional cost $I_T$ or not.

### III. Equilibrium

The parameters in the model may have various values and there would be multiple equilibria depending on the values of the parameters. We will just show that for some values of the parameters there exists an equilibrium where a high type investor continues to invest the additional cost $I_H$ even after the current investment turns out to be unprofitable at the beginning of $t = 2$. Then, we will analyze characteristics of that specific equilibrium.

We will show that there exists an equilibrium as follows. In this equilibrium a high type investor will invest $S$ in $t = 1$ if and only if the signal, $\hat{I}_H$ is less than or equal to a certain level, $\hat{I}_H^* (\hat{I}_H \leq \hat{I}_H^*)$. We
will calculate the exact value of $\hat{I}_{H}^{**}$ later. A low type investor will also invest $S$ in $t = 1$ if and only if $\hat{I}_{L} \leq \hat{I}_{L}^{**}$. Once $S$ is invested, a high type investor will always invest $I$ additionally in $t = 2$. The high type investor will keep investing in $t = 2$ when the additional cost turns out to be bigger than expected in order to protect the reputation.

On the other hand, a low type investor will invest only when the additional cost is same as the signal, $I_{L} = \hat{I}_{L}$. The beliefs in the equilibrium will be $\mu_{S} = 0$ and $\mu_{I} > \mu_{N} > 0$ which we will show in the following proof.

We will prove the existence of this equilibrium in the following proposition 1.

**Proposition 1**: For some $k$ there exists an equilibrium where a high type investor who has already invested in the sunk cost $S$ will always invest in the additional cost $I_{H}$ for the purpose of future investment opportunities, even though the current investment itself turns out to be unprofitable.

Proof) Let $k = \mu_{S} - \hat{I}_{H} + \epsilon, (\epsilon > 0)$ and consider the case where $S$ is already invested in $t = 1$. If the signal turns out to be incorrect in $t = 2$, and if the investor does not care about reputation, the investor will not invest in $t = 2$ because $\pi_{S} - \hat{I}_{H} - k < 0$. However, the investor has to consider the reputation, $A(\mu)$, and will compare $\pi = \pi_{S} - \hat{I}_{H} - k + A(\mu_{I})$ with $\pi = A(\mu_{2})$. We will soon show that $\mu_{I} > \mu_{S} = 0$ in this equilibrium. Therefore, for $\epsilon$ small enough to satisfy $\epsilon < A(\mu_{I}) - A(0) = A(\mu_{I})$, a high type investor will choose to invest in $t = 2$ since $\pi_{S} - \hat{I}_{H} - k + A(\mu_{I}) > A(\mu_{S})$. As we have assumed, a low type investor will invest in $t = 2$ if and only if the signal turns out to be correct.

Now let’s consider the investor’s decision in $t = 1$.

The necessary and sufficient condition for a high type to invest $S$ in $t = 1$ is as follows.
If we define \( \hat{I}_H^{**} = - S + \pi_S - (1 - \alpha_H)k + A(\mu_I) - A(\mu_N) \), equation (1) can be rewritten as \( \hat{I}_H \leq \hat{I}_H^{**} \).

We will define the probability that a high type will invest \( S \) in \( t = 1 \) as \( \theta_H \equiv \text{prob}(\hat{I}_H \leq \hat{I}_H^{**}) \).

Through a similar process one can easily see that the necessary and sufficient condition for a low type investor to invest in \( S \) is as follows.

\[
-S + \alpha_L(\pi_S - \hat{I}_H) + \alpha_L A(\mu_I) + (1 - \alpha_L) A(\mu_S) - A(\mu_N) \geq 0
\]

(2)

Once we define \( \hat{I}_L^{**} \equiv \frac{1}{\alpha_L} \left\{ - S + \alpha_L(\pi_S) + \{ \alpha_L A(\mu_I) + (1 - \alpha_L) A(\mu_S) - A(\mu_N) \} \right\}, \) we can rewrite equation (2) as \( \hat{I}_L \leq \hat{I}_L^{**} \).

Also, we will define the probability that a low type will invest \( S \) in \( t = 1 \) as \( \theta_L \equiv \text{prob}(\hat{I}_L \leq \hat{I}_L^{**}) \).

Because the high type investor will always invest in \( t = 2 \) in this equilibrium, it is obvious that \( \mu_S = 0 \) and \( A(\mu_S) = 0 \). The other beliefs will be

\[
\mu_I = \frac{\gamma \theta_H}{\gamma \theta_H + (1 - \gamma) \theta_L \alpha_L} \quad \text{and} \quad \mu_N = \frac{\gamma(1 - \theta_H)}{\gamma(1 - \theta_H) + (1 - \gamma)(1 - \theta_L)}.
\]

If we set \( \hat{I}_H = \hat{I}_L^{**} \), this \( \hat{I}_H \) satisfies equation (1) with a strict inequality.

\[
-S + \alpha_H(\pi_S - \hat{I}_H^{**}) + (1 - \alpha_H)(\pi_S - \hat{I}_L^{**} - k - A(\mu_I)) + \alpha_H A(\mu_I) - A(\mu_N) > 0
\]

(3)

We see a strict inequality in (3) because \( - S + \alpha_L(\pi_S - \hat{I}_L^{**}) + \alpha_L A(\mu_I) - A(\mu_N) = 0, \quad \alpha_H A(\mu_I) > \alpha_L A(\mu_I), \quad \text{and} \quad \pi_S - \hat{I}_L^{**} - k + A(\mu_I) > 0. \)

This implies that \( \hat{I}_H^{**} > \hat{I}_L^{**} \) and \( \theta_H > \theta_L \).

From \( \theta_H > \theta_L \) we can easily calculate that \( \mu_I > \mu_N \). Therefore, there exists an equilibrium where a high type investor will invest in \( t = 2 \) even when it becomes clear that the current investment would be unprofitable. Q.E.D.
In our model there can be a situation where the investor who expected a profitable return when he invests $S$ in $t = 1$ but later found out that the investment would be unprofitable at the beginning of $t = 2$. If the sunk cost should truly be forgotten, the investor should stop investing the additional cost. However, if a reputation for future investment opportunities is at stake, the investor may continue to invest even though a continuing investment would only increase the loss furthermore.

The following proposition 2 is on how this consideration on the reputation would influence the investor’s decision on the sunk cost in $t = 1$.

In order to understand proposition 2, let’s make one more definition. We have already defined $\hat{I}_H^{**}$ as the cut-off level based on which the investor decides whether to invest the sunk cost in $t = 1$ or not, when reputation matters. Now, let’s define $\hat{I}_H^*$ as the cut-off level based on which the investor decides whether to invest the sunk cost in $t = 1$ or not, when reputation does not matter.

**Proposition 2**: For $\alpha_H$ large enough a high type investor is more likely to invest the sunk cost in $t = 1$ when reputation matters, $\hat{I}_H^{**} > \hat{I}_H^*$. On the other hand, for $\alpha_H$ small enough and for $k$ large enough a high type investor is less likely to invest the sunk cost in $t = 1$ when reputation matters, $\hat{I}_H^{**} < \hat{I}_H^*$.

Proof) First of all, when the reputation does not matter, the necessary and sufficient condition for the investor to invest $S$ in $t = 1$ is as follows.

$$-S + \alpha_H (\pi_S - \hat{I}_H) \geq 0$$

(4)

As a result, the investor will invest $S$ in $t = 1$ if and only if
\[ \hat{I}_H \leq \hat{I}_H^* \equiv \frac{1}{\alpha_H} [-S + \alpha_H \pi_S]. \]

If we substitute \( \hat{I}_H^* \) for \( \hat{I}_H \) on the left side of equation (1), we will get

\[
-S + (\pi_S - \hat{I}_H^*) - (1 - \alpha_H) k + A(\mu_I) - A(\mu_N) \\
= (1 - \alpha_H)(\pi_S - \hat{I}_H^* - k + A(\mu_I)) + \{\alpha_H A(\mu_I) - A(\mu_N)\} \tag{5}
\]

If equation (5) is bigger than 0, it will be \( \hat{I}_H^* \) > \( \hat{I}_H^* \). If equation (5) is smaller than 0, it will be \( \hat{I}_H^{**} < \hat{I}_H^* \). Since \( \pi_S - \hat{I}_H^* - k + A(\mu_N) > 0 \) by assumption, equation (5) will have a positive value if \( \alpha_H A(\mu_I) - A(\mu_N) > 0 \). We have already shown that \( \mu_I > \mu_N \). Therefore, if \( \alpha_H \) is large enough we will get \( \alpha_H A(\mu_I) - A(\mu_N) > 0 \) and \( \hat{I}_H^{**} > \hat{I}_H^* \). On the other hand, if \( \alpha_H \) is small enough so that \( (1 - \alpha_H) (\pi_S - \hat{I}_H^* - k + A(\mu_I)) < \epsilon \) for some small \( \epsilon \) and if \( k \) is large enough so that \( \{\alpha_H A(\mu_I) - A(\mu_N)\} < -2\epsilon \), equation (5) will have a negative value and we will get \( \hat{I}_H^* < \hat{I}_H^* \). Q.E.D.

In proposition 2, \( \hat{I}_H^{**} > \hat{I}_H^* \) means that the investor is more likely to invest \( S \) in \( t = 1 \) when the reputation matters. On the other hand, \( \hat{I}_H^{**} < \hat{I}_H^* \) means that the investor is less likely to invest \( S \) in \( t = 1 \) when the reputation matters. These results are rather intuitive. If \( \alpha_H \) is relatively large, the current investment is more likely to be profitable when the investor gets a good signal in \( t = 1 \). Therefore, if the reputation matters, it will be additionally beneficial to the investor. On the other hand, if \( \alpha_H \) is relatively small, the current investment is less likely to be profitable even though the investor gets a good signal in \( t = 1 \). The investor in \( t = 2 \) has to invest the additional investment \( I \) if the reputation matters even if the investment is unprofitable. Because of this possibility that the investor may have to continue an unprofitable investment, the investor may become a bit reluctant to invest the sunk cost in \( t = 1 \).
Ⅳ. Conclusion

The economic theory says that bygones are bygones. Therefore, an investor should stop the project when it turns out to be a failure, even if the initial investment cannot be recovered.

However, if the current investment project is related to the reputation of the investor, the investor may have to continue the investment in spite of the expected loss to protect reputation and future opportunities.

On the other hand, it is conceivable that an investor who concerns about reputation may become less enthusiastic on investment opportunities because he cannot stop the continuing investment even after it becomes clear that the investment will be unprofitable.

What seems to be an unreasonable response towards failure in the real world may be explained as a very reasonable response. This research is part of this line of effort.
References


