Collateral Constraints, Sticky Wages, and Monetary Policy

Kwang Hwan Kim* · Joonseok Oh**

This paper investigates the role of collateral constraints in the transmission of monetary policy shocks in a two-sector sticky price general equilibrium model with nondurable and durable goods. While many researchers have stressed the role of collateral constraints in a one-sector sticky price model, the role of such constraints in two-sector sticky price models has been relatively underexplored. This study shows that nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable spending in the two-sector sticky price general equilibrium model.

Keywords: Durable goods, Collateral constraints, Sticky nominal wages
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I. Introduction

Durable goods play a significant role in the discussion of monetary policy. According to the empirical results documented by Erceg and Levin (2006), Barsky et al. (2007), and Monacelli (2009), durable spending responds procyclically to monetary policy. An important characteristic of durable goods is that households use the durable goods as collateral. Recently, residential mortgage loans have accounted for a great part of total household debt for multiple countries. For example, in South Korea, the proportion of debt accounted by mortgage loans has increased since 2007. About 61% of loans to household of depository corporations consist of mortgage loans in 2012.1) Moreover, in the United States, collateralized debt represents 88.6% of total household debt in 2010.2)

The role of borrowing constraints at the household level in a one-sector model has been widely stressed by Zeldes (1989), Jappelli and Pagano (1989), and Campbell and Mankiw (1989). While these papers have stressed the importance of financial frictions for macroeconomic fluctuations, they are based on a partial equilibrium model. Chah et al. (1995) develop and test a theory of optimal consumption behavior in the presence of collateral constraints. Campbell and Hercowitz (2006) study the role of collateralized household debt in macroeconomic stabilization in a real business cycle model. Unlike all these previous studies, this paper investigates the role of collateral constraints in explaining the procyclical response of durable spending to monetary policy in a two-sector sticky price general equilibrium model.

Our paper is not the first paper to investigate the role of collateral constraints in a two-sector sticky price general equilibrium model. Iacoviello (2005), Monacelli (2009), Sterk (2010), and Iacoviello and Neri (2010) also analyze how collateral constraints work in the transmission of monetary policy in a two-sector price model. However, Iacoviello (2005) focuses on the

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1) Relevant data is available at the Economic Statistics System of the Bank of Korea.
2) 2012 Survey of Consumer Finances
nondurable goods with durable stock fixed. In Iacoviello and Neri (2010), the role of collateral constraints is not the main issue; their study focuses on showing that housing market spillovers concentrate on consumption rather than business investment. Monacelli (2009) and Sterk (2010) reach opposite conclusions. Monacelli (2009) claims that the introduction of collateral constraints accelerates the procyclical response of durable spending to monetary policy shocks in a standard two-sector sticky price model. However, Sterk (2010) demonstrates that Monacelli (2009) neglects general equilibrium effects. The changes in borrowing and durable purchases by the borrowers have an influence on the savers. Thus, collateral constraints rather decelerate the response of durable spending to monetary policy shocks in the general equilibrium model.

This paper shows that whether collateral constraints at the household level accelerate or decelerate durable spending to monetary policy shocks depends on how the labor market is modeled. We find that the degree of nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable spending in a two-sector sticky price general equilibrium model. Monacelli (2009) and Sterk (2010) assume Walrasian labor markets. However, if there exits nominal wage rigidity due to wage adjustment costs, the nominal wage responds sluggishly to shocks, generating inefficient variations in the wage markup. In addition, wage inflation, combined with the staggering of wage adjustments, affects the allocation of labor. Since wage inflation responds more greatly to monetary policy shocks in the existence of collateral constraints, the effects of the shocks on durable spending are accelerated in the model with sticky nominal wages.

Although labor market rigidity enables collateral constraints to accelerate the procyclical response of durable spending in the initial period, the sensitivity of durables to monetary policy shocks is excessively larger than that of nondurables, which is at odds with empirical data. Since sticky nominal wages lead to the rapid over-shooting of durable spending, as Carlstrom and Fuerst (2010) shows, this paper additionally considers adjustment costs in durable
investment in order to smooth production over time.

The rest of the paper is organized as follows. Section 2 outlines the two-sector sticky price general equilibrium model incorporated with collateral constraints and nominal wage rigidity. Section 3 describes the parameter calibration and solution method. Section 4 presents the analytical discussion of the model in regards to monetary policy shocks. Section 5 investigates the numerical results of the model. Section 6 incorporates durable investment adjustment costs in the model. Section 7 concludes.

II. The Model

This study analyzes a two-sector sticky price model with nondurable and durable goods in order to investigating the role of collateral constraints. We add nominal wage rigidity to the standard two-sector sticky price model with collateral constraints (as in Monacelli (2009)).

1. Households

The model introduces two types of households with different time preference rates, meaning that one group of households is more patient than the other. They are entitled borrowers and savers respectively because the impatient households borrow from the patient households in equilibrium. The borrowers are confronted with collateral constraints. We suppose that there is a continuum of agents of measure 1 in each of the two groups. The size of the total population is normalized to 1 and the weight of the borrowers is set to \( \omega \).

1) Borrowers

Borrowers have a discount factor \( \beta \). They maximize the following utility program:
\[ E_0 \sum_{t=0}^{\infty} \beta^t U^b(X_t^b(i), N_t^b(i)) \]  

where \( U^b(\cdot) \) is a utility function depending on labor supply \( N_t^b \) and a CES consumption basket \( X_t^b \) that consists of nondurable goods \( C_t^b \) and the stock of durable goods \( D_t^b \):

\[ U_b(X_t^b(i), N_t^b(i)) = \log X_t^b(i) - \frac{\nu^b N_t^b(i)^{1 + \phi^b}}{1 + \phi^b} \]  

\[ X_t^b(i) = \left[ (1 - \alpha)^\eta C_t^b(i) + \alpha^\eta D_t^b(i) \right]^{\frac{\eta}{\eta - 1}} \]

where \( \nu^b \) is the parameter that indexes the preference for hours worked of borrowers, \( \phi^b \) is the inverse elasticity of borrowers' labor supply, \( \alpha > 0 \) is the share of durable goods in the CES consumption basket, and \( \eta > 0 \) is the elasticity of substitution between services of nondurable and durable goods.

The borrowers face the following budget constraint and collateral constraint in real terms (in units of nondurables), and the law of motion for durable goods:

\[ C_t^b(i) + q_t I_t^b(i) + R_t \frac{b_{t-1}(i)}{\pi_{c,t}} + A C_{w,t}^b(i) = b_t(i) + u_t^b(i) N_t^b(i) \]  

\[ R_t b_t(i) \leq (1 - \chi)(1 - \delta) E_t(D_t^b(i)) q_{t+1} \pi_{c,t+1} \]  

\[ I_t^b(i) = D_t^b(i) - (1 - \delta) D_{t-1}^b \]

where \( q_t = \frac{P_{d,t}}{P_{c,t}} \) is the relative price of the durable goods, \( I_t^b(i) \) is durable investment as the flow variable, \( b_t(i) \) is end-of-period \( t \) real one-period debt, \( R_{t-1} \) is the gross nominal interest rate on loan contracts stipulated at time \( t - 1 \), \( \pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}} \) is nondurable goods inflation, \( A C_{w,t}^b(i) \) is the real cost
of adjusting nominal wages, \( w^b_t(i) = \frac{W^b_t(i)}{P_{e,t}} \) is the real wage for the borrower, \( \chi \) is the fraction of durables that cannot be used as collateral, and \( \delta \) is the depreciation rate for durable goods. This study is based on the assumption that the collateral constraints is always binding: in a neighborhood of the steady state, equation (5) is always satisfied with equality. Also, labor is assumed to be perfectly mobile across sectors, implying that the real wage rate is common across sectors.

We follow Kim (2000) by modeling nominal wage rigidity through the cost of adjusting nominal wages. The wage adjustment cost function is assumed to be quadratic and zero at the steady state. The real total adjustment cost for a borrower \( i \) is given by

\[
AC^b_{w,t}(i) = \frac{\phi^b_w}{2} \left( \frac{W^b_t(i)}{W^b_{t-1}(i)} - 1 \right)^2 w^b_t
\]

where \( \phi^b_w \) is the wage adjustment cost parameter.

Let \( \lambda^b_t(i), \lambda^b_t(i)\psi_t(i) \) be defined as the multipliers on constraints (4) and (5) respectively. We assume a symmetric equilibrium to rule out potential multiple equilibria in which identical agents behave differently because of different initial conditions. In a symmetric equilibrium, each borrower is endowed with the same initial condition and makes identical decisions so that \( C^b_t(i) = C^b_t, \quad D^b_t(i) = D^b_t, \quad I^b_t(i) = I^b_t, \quad b_t(i) = b_t, \quad \lambda^b_t(i) = \lambda^b_t, \) and \( \psi_t(i) = \psi_t \). The first-order conditions are as follows:

\[
U^b_{c,t} = \lambda^b_t
\]

\[
q_t U^b_{c,t} = U^b_{d,t} + \beta(1 - \delta)E_t(q_{t+1} U^b_{c,t+1}) + (1 - \chi)(1 - \delta)E_t(q_t U^b_t \psi_t \pi_{d,t+1})
\]

\[
R_t \psi_t = 1 - \beta \left( \frac{\lambda^b_{t+1}}{\lambda^b_t} \frac{R_t}{\pi_{c,t+1}} \right)
\]
where \( U_{c,t}^b \) and \( U_{d,t}^b \) are the marginal utilities of nondurables and durables respectively.

In equation (8), the borrowers' marginal utility of nondurable consumption is equated to the shadow value of budget constraints (4). Equation (9) requires the borrowers to equate the marginal utility of nondurable consumption to the shadow value of durables. In the right hand side, the last term is proportional to \( \psi_t \), which measures the tightness of the borrowing constraints. Equation (10) is a modified version of the standard Euler equation. It would reduce to the standard Euler equation if the collateral constraint is not binding, that is, when \( \psi_t = 0 \) for all \( t \).

2) Savers

Savers have a discount factor \( \gamma \). The key feature is that \( \gamma > \beta \). This is because the savers are more patient than the borrowers. It is assumed that the savers are the owners of the intermediate goods firms in each sector. They maximize the following utility program:

\[
E_0 \sum_{t=0}^{\infty} \gamma^t U^s(X_t^s(i), N_t^s(i))
\]

where \( U^s(\cdot) \) is a utility function depending on labor supply \( N_t^s \) and a CES consumption basket \( X_t^s \) that consists of nondurable goods \( G_t^s \) and the stock of durable goods \( D_t^s \):

\[
U^s(X_t^s(i), N_t^s(i)) = \log X_t^s(i) - \frac{\nu^s N_t^s(i)^{1+\varphi^s}}{1 + \varphi^s}
\]

\[
X_t^s(i) = \left[ (1 - \alpha)^\eta C_t^s(i) + \alpha^\eta D_t^s(i) \right]^\frac{\eta}{\eta - 1}
\]
where $\nu^s$ is the parameter that indexes the preference for hours worked of savers, and $\varphi^s$ is the inverse elasticity of savers' labor supply.

The savers face the following budget constraint in real terms (in units of nondurables) and the law of motion for durable goods:

$$C_t^s(i) + q_t I_t^s(i) + s_t(i) + AC_{w,t}^s(i)$$

$$= R_t \frac{s_{t-1}(i)}{\pi_{c,t}} + w_t^s(i) N_t^s(i) + (1 - \omega)^{-1} \Pi_t(i)$$

$$I_t^s(i) = D_t^s(i) - (1 - \delta) D_{t-1}^s$$

(14)

(15)

where $I_t^s(i)$ is durable investment as the flow variable, $s_t(i)$ is end-of-period $t$ real one-period savings, $AC_{w,t}^s(i)$ is the real cost of adjusting nominal wages, and $\Pi_t(i)$ is aggregate nominal profit faced by the monopolistic competitive firms.

Similarly, the wage adjustment cost function is assumed to be quadratic and zero at the steady state. The real total adjustment cost for saver $i$ is given by

$$AC_{w,t}^s(i) = \frac{\phi_w^s}{2} \left( \frac{W_t^s(i)}{W_{t-1}^s(i)} - 1 \right)^2 w_t^s$$

(16)

where $\phi_w^s$ is the wage adjustment cost parameter.

Let $\lambda_t^s(i)$ be defined as the multiplier on the constraint (14). Similar to the borrowers, we suppose a symmetric equilibrium. Hence, the first order conditions of the savers are as follows:

$$U_{c,t}^s = \lambda_t^s$$

$$q_t U_{c,t}^s = U_{d,t}^s + \gamma (1 - \delta) E_t (q_{t+1} U_{c,t+1}^s)$$

(17)

(18)
where \( U_{c,t}^s \) and \( U_{d,t}^s \) are the marginal utilities of nondurables and durables respectively.

In equation (17), the savers' marginal utility of nondurable consumption is equated to the shadow value of budget constraints (14). Equation (18) requires that the savers equate the marginal utility of nondurable consumption to the shadow value of durables. Equation (19) is the standard Euler equation.

2. Employment Unions

Borrowers and savers supply homogeneous labor services to the employment unions. We assume that there are two unions, one for each household pair. The unions offer differentiated labor services to intermediate goods firms according to

\[
N_t^b = \left( \int_0^1 N_t^b(i) \frac{\epsilon^b_{w} - 1}{\epsilon^b_{w}} \, ds \right)^{\frac{\epsilon^b_{w}}{\epsilon^b_{w} - 1}}
\]

(20)

\[
N_t^s = \left( \int_0^1 N_t^s(i) \frac{\epsilon^s_{w} - 1}{\epsilon^s_{w}} \, ds \right)^{\frac{\epsilon^s_{w}}{\epsilon^s_{w} - 1}}
\]

(21)

Here, \( N_t^b \) and \( N_t^s \) are the amounts of the composite labor supplied by the aggregators, \( N_t^b(i) \) and \( N_t^s(i) \) are the labor supplies of household \( i \) and \( \epsilon^b_{w} \) and \( \epsilon^s_{w} \) are the elasticities of substitution among labor varieties of each household.

The first-order conditions for profit maximization yield the demand functions of each household's differentiated labor:
where $W^b_t(i)$ and $W^s_t(i)$ are the nominal wages of each household $i$ respectively and $W^b_t$ and $W^s_t$ are the aggregate nominal wage indexes. The labor demand by firms serves as another constraint facing each household.

Each household chooses $W^b_t(i)$ and $W^s_t(i)$ in order to maximize the utility program above subject to its respective demand function as well as budget constraint. Each household is endowed with the same initial condition and makes identical decisions. This is why we prefer the Rotemberg mechanism to the Calvo mechanism to avoid heterogeneity in wages: this way all the agents earn the same wages. Therefore, the first-order conditions are as follows:

$$\phi^b_w, (\pi^b_{w,t} - 1) \pi^b_{w,t}$$

$$= \phi^b_w \beta E_t \lambda^b_{t+1} \left[(\pi^b_{w,t+1} - 1) \pi^b_{w,t+1} \frac{w^b_{t+1}}{w^b_t} \right] + (1 - \epsilon^b_w) N^b_t + \epsilon^b_w MRS^b_t \lambda^b_{t+1} \frac{N^b_t}{w^b_t}$$

(24)

$$\phi^s_w, (\pi^s_{w,t} - 1) \pi^s_{w,t}$$

$$= \phi^s_w \gamma E_t \lambda^s_{t+1} \left[(\pi^s_{w,t+1} - 1) \pi^s_{w,t+1} \frac{w^s_{t+1}}{w^s_t} \right] + (1 - \epsilon^s_w) N^s_t + \epsilon^s_w MRS^s_t \lambda^s_{t+1} \frac{N^s_t}{w^s_t}$$

(25)

where $MRS^b_t$ and $MRS^s_t$ are borrowers' and savers' marginal rates of substitution between leisure and consumption respectively.

The following New Keynesian Wage Phillips curves can be obtained by log-linearizing Equation (24) and Equation (25):

$$N^b_t(i) = \left( \frac{W^b_t(i)}{W^b_t} \right)^{-\epsilon^b_w} N^b_t$$

(22)

$$N^s_t(i) = \left( \frac{W^s_t(i)}{W^s_t} \right)^{-\epsilon^s_w} N^s_t$$

(23)
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\[ \pi_{w,t}^b = \beta E_t \pi_{w,t+1}^b + \frac{(\epsilon_w^b - 1)N_b}{\phi_w^b} (MRS_t^b - \tilde{\pi}_t^b) \]  (26)

\[ \pi_{w,t}^s = \gamma E_t \pi_{w,t+1}^s + \frac{(\epsilon_w^s - 1)N_s}{\phi_w^s} (MRS_t^s - \tilde{\pi}_t^s) \]  (27)

where \( N^b \) and \( N^s \) are steady state labor supplies of each household.

3. Firms

This paper assumes the existence of a continuum of monopolistically competitive firms, indexed by \( s \in [0, 1] \), producing differentiated intermediate goods in each sector. A final good in each sector is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods and sells it to the households.

1) Final goods firms

The final goods in each sector are aggregated by the constant elasticity of substitution (CES) technology:

\[ Y_{j,t} = \left( \int_0^1 Y_{j,t}(s) \left( \frac{\epsilon_j - 1}{\epsilon_j} \right)^{\frac{\epsilon_j}{\epsilon_j - 1}} ds \right)^{\frac{\epsilon_j}{\epsilon_j - 1}} \]  (28)

Let \( Y_{j,t}(s) \) denote the inputs of intermediate good \( s \) in sector \( j = c, d \) and let \( Y_{j,t} \) be the quantity of the final good produced.

Profit maximization delivers the following equation for demand for intermediate goods:

\[ Y_{j,t}(s) = \left( \frac{P_{j,t}(s)}{P_{j,t}} \right)^{-\epsilon_j} Y_{j,t} \]  (29)
where \( P_{j,t}(s) \) is the price of intermediate good \( s \) and \( P_{j,t} \) is the aggregate price level in sector \( j \). Finally, the zero-profit condition implies that:

\[
P_{j,t} = \left( \int_0^1 P_{j,t}(s)^{1-\varepsilon_j} \, ds \right)^{1/(1-\varepsilon_j)}
\]

(30)

2) Intermediate goods firms

Intermediate goods firms are monopolistically competitive. Inputs for the final goods are supplied by intermediate goods firms. Firm \( s \) in sector \( j \) has the following production function:

\[
Y_{j,t}(s) = N_{j,t}(s)
\]

(31)

where \( N_{j,t}(s) \) denotes the amount of labor that is used by firm \( s \) in sector \( j \) at period \( t \). Intermediate goods firms have some price-setting power because intermediate goods are slightly differentiated. Nominal profit is as follow:

\[
\Pi_{j,t} = P_{j,t}(s) Y_{j,t}(s) - W_t N_{j,t}(s)
\]

(32)

Intermediate goods firms are assumed to set nominal prices in a staggered fashion according to the stochastic time-dependent rule proposed by Calvo (1983). Each firm in sector \( j = c, d \) resets its price with the probability of \( 1 - \theta_j \) each period. Thus, a fraction \( \theta_j \) of firms cannot re-optimize their prices.

An intermediate goods firm resetting its price in period \( t \) in sector \( j = c, d \) will seek to maximize the profits using the nominal stochastic factor of the savers:

\[
E_t \sum_{k=0}^{\infty} \theta_j^k \gamma^k \frac{\lambda_t^{s+k}}{\lambda_t^s} \frac{P_{c,t}}{P_{c,t+k}} \Pi_{j,t+k}
\]

(33)
subject to Equation (29) and Equation (31). The optimal reset prices, $P^*_{j,t}$, are as follows:

$$P^*_{c,t} = \frac{\varepsilon_c E_t \sum_{k=0}^{\infty} \theta^k \lambda^*_{c,t+k} W_{t+k} P^e_{c,t+k} Y_{c,t+k}}{(\varepsilon_c - 1) E_t \sum_{k=0}^{\infty} \theta^k \lambda^*_{c,t+k} P^e_{c,t+k} Y_{c,t+k}}$$ (34)

$$P^*_{d,t} = \frac{\varepsilon_d E_t \sum_{k=0}^{\infty} \theta^k \lambda^*_{d,t+k} W_{t+k} P^e_{d,t+k} Y_{d,t+k}}{(\varepsilon_d - 1) E_t \sum_{k=0}^{\infty} \theta^k \lambda^*_{d,t+k} P^e_{d,t+k} Y_{d,t+k}}$$ (35)

Finally, the equation describing the dynamics for the aggregate price level in sector $j = c, d$ is given by

$$P_{j,t} = \left[ (1 - \theta_j) (P^*_{j,t})^{1-\varepsilon_j} + \theta_j P_{j,t-1} \right]^{1/(1-\varepsilon_j)}$$ (36)

By log-linearizing around a sectoral zero-inflation steady state, Equation (34), Equation (35), and Equation (36) take the form of the forward looking New Keynesian Phillips curve for each sector:

$$\hat{\pi}_{c,t} = \gamma E_t \hat{\pi}_{c,t+1} + \frac{(1 - \theta_c)(1 - \gamma \theta_c)}{\theta_c} (\hat{\omega}_t - \hat{\theta}_t)$$ (37)

$$\hat{\pi}_{d,t} = \gamma E_t \hat{\pi}_{d,t+1} + \frac{(1 - \theta_d)(1 - \gamma \theta_d)}{\theta_d} (\hat{\omega}_t - \hat{\theta}_t)$$ (38)

4. Monetary Policy

To facilitate comparison with Sterk (2010), it is assumed that monetary policy is conducted by means of a simple Taylor-type rule:

$$\frac{R_t}{R} = \left( \frac{\pi_t}{\overline{\pi}} \right)^{\phi_R} \exp(\varepsilon_t)$$ (39)
where \( \pi_t = \pi_{c,t}^{1-\tau} \pi_{d,t}^{\tau} \) is a composite inflation index and \( R \) and \( \pi \) are the steady state levels of the gross nominal interest rate and the inflation index, respectively. The monetary policy shock, \( \varepsilon_t \), evolves according to

\[
\exp(\varepsilon_t) = \exp(\varepsilon_{t-1})^\rho u_t
\]

with \( u_t \sim iid \) and \( 0 < \rho < 1 \).

5. Market Clearing

Good market clearing requires that the production function of the final good equal to total expenditure across all households:

\[
Y_{c,t} = \omega C_t^b + (1 - \omega) C_t^s
\]
\[
Y_{d,t} = \omega I_t^b + (1 - \omega) I_t^s
\]

where \( Y_{j,t} = \int Y_{j,t}(s)ds \).

The economy-wide total output \( Y_t \) is given by:

\[
Y_t = Y_{c,t} + q Y_{d,t}
\]

where \( q \) is the deterministic steady-state value of the relative price of durable goods \( (q = 1) \).

Bond market and labor market clearing requires:

\[
\omega b_t = (1 - \omega) s_t
\]
\[
N_{c,t} + N_{d,t} = \omega N_t^b + (1 - \omega) N_t^s
\]

where \( N_{j,t} = \int N_{j,t}(s)ds \).
Since we assume that there is no difference between the labor supply of borrowers and savers, the wage for borrowers is identical to that for savers:

\[ w^b_t = w^s_t = w_t \quad \text{and} \quad \pi^b_{w,t} = \pi^s_{w,t} = \pi_{w,t} \]  \hspace{1cm} (46)

III. Calibration and Solution Method

The models are calibrated to a quarterly frequency and the calibration follows Monacelli (2009). Table 1 provides a summary of the key parameters. The parameters \( \nu^b \) and \( \nu^s \) are calibrated to normalize labor supply to 1 in the steady state for each type of households. The parameter \( \alpha \) in the CES basket is the same for both borrowers and savers, and it is calibrated such that in the steady state, aggregate durable spending accounts for 20% of aggregate expenditure. The parameter \( \tau \) reflecting the weight of durable goods in the composite inflation index in the monetary policy rule is set to 0. In other words, monetary policy responds to nondurable inflation only. 3)

This paper imposes the Calvo parameter in the nondurable sector to correspond to four quarter stickiness \( (\theta_c = 0.75) \). Since the degree of durable price rigidity has important influences on the model dynamics, the Calvo parameter of durable sector \( (\theta_d) \) is considered from full flexibility to four quarter stickiness. In addition, we set \( \omega^b_w = 260.08 \) and \( \omega^s_w = 266.02 \) so that the value of the Calvo parameter for wage rigidity is about 0.70. This paper log-linearizes the model around the deterministic steady state and solves the recursive law of motion.

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3) The results of Monacelli (2009) and Sterk (2010) are based on a value of \( \tau \) is set to 0. Even if adopting the assumption that monetary policy also responds to durable inflation \( (\tau = 0.2) \), this will not affect our results discussed below.
## Table 1: Baseline Quarterly Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\omega$</td>
<td>Share of borrowers</td>
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<tr>
<td>$\beta$</td>
<td>Discount factor for borrowers</td>
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<td>$\gamma$</td>
<td>Discount factor for savers</td>
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<td>$\delta$</td>
<td>Depreciation rate of durable goods</td>
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<td>$\chi$</td>
<td>Fraction of durables that cannot be used as a collateral</td>
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<td>$\eta$</td>
<td>Elasticity of substitution between $C_i$ and $D_i$</td>
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<tr>
<td>$\varphi^b$</td>
<td>Inverse elasticity of labor supply of borrowers</td>
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<tr>
<td>$\varphi^s$</td>
<td>Inverse elasticity of labor supply of savers</td>
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<tr>
<td>$\varepsilon_c$</td>
<td>Elasticity of substitution among nondurable varieties</td>
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<td>$\varepsilon_d$</td>
<td>Elasticity of substitution among durable varieties</td>
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<td>$\varepsilon_w^s$</td>
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<td>$\theta_c$</td>
<td>Calvo parameter for nondurable goods price</td>
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<tr>
<td>$\theta_d$</td>
<td>Calvo parameter for durable goods price</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi^b_w$</td>
<td>Wage adjustment cost parameter for borrowers</td>
<td>260.08</td>
</tr>
<tr>
<td>$\phi^s_w$</td>
<td>Wage adjustment cost parameter for savers</td>
<td>266.02</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Weight of durables in the composite inflation index</td>
<td>0.00</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Coefficient on inflation in monetary policy rule</td>
<td>1.50</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence parameter monetary policy shocks</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: As a robustness check, $1/\varphi^b$ and $1/\varphi^s$ was set to the values [2, 4] used by macroeconomists for the model. This did not impact on our results.
IV. Analytical Discussions

In this section, if the prices of durable goods are flexible, we analytically demonstrate that the response of a two-sector sticky price model to monetary policy shocks depends on how the labor market is designed. This paper shows that nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable spending.

1. Sticky Nominal Wages and Collateral Constraints

This study investigates the role of collateral constraints in a two-sector sticky price general equilibrium model with nominal wage rigidity. To begin with, it is necessary to express aggregate output and aggregate nondurable consumption as functions of the shadow values of durables. In this paper, $V_{d,t}^{b}$ and $V_{d,t}^{s}$ are defined as the shadow values of durables for the borrowers and the savers respectively, and their durables optimality conditions (9) and (18) can be rewritten as follows:

$$V_{d,t}^{b} = q_t U_{c,t}^{b} = \frac{U_{d,t}^{b} + \beta(1 - \delta)E_t(V_{d,t+1}^{b})}{1 - (1 - \chi)(1 - \delta)\psi_t E_t(\pi_{d,t+1})} \quad (47)$$

$$V_{d,t}^{s} = q_t U_{c,t}^{s} = \frac{U_{d,t}^{s} + \gamma(1 - \delta)E_t(V_{d,t+1}^{s})}{1 - (1 - \delta)\psi_t E_t(\pi_{d,t+1})} \quad (48)$$

Recall that $V_{d,t}^{b}$ increases when the collateral constraints tightens ($\psi_t$ increases). On the other hand, $V_{d,t}^{s}$ is quasi-constant.

In the model, by using New Keynesian Wage Phillips curves (26) and (27), production function (31), and labor market clearing condition (45), the log-approximation of the aggregate output $\hat{Y}_t$ can be expressed as follows:
The wage inflation effect and wealth effect mainly have influence on aggregate output. Wage inflation effects, \( \frac{\phi^b_w}{(\varepsilon^b_w - 1)N^b}(\hat{\pi}_{w,t} - \beta E_{t}\hat{\pi}_{w,t-1}) \) and \( \frac{\phi^s_w}{(\varepsilon^s_w - 1)N^s}(\hat{\pi}_{w,t} - \gamma E_{t}\hat{\pi}_{w,t-1}) \), consist of the current wage inflation and the discounted expected future wage inflation. When current wage inflation is higher than the discounted expected future one, this effect has a positive influence on labor allocation. Also, this effect has a larger impact on labor allocation as the degree of wage stickiness is higher. Since we calibrate \( \phi^b_w = 260.08 \) and \( \phi^s_w = 266.02 \), the wage inflation effect is much more important to the dynamics in this model than wealth effect. \(<Figure 1>\) plots the IRFs for wage inflation to monetary tightening for the sticky nominal wages model. \(<Figure 1>\) shows that the wage inflation effects have a negative impact on labor after monetary tightening.

Aggregate output decreases in the model including sticky nominal wages with collateral constraints after monetary tightening. If durable goods prices are flexible, the real wage in units of durables \( \hat{w}_t - \hat{q}_t \) is 0 because durable prices are set according to a constant markup over the nominal wage. The shadow value of durables for savers \( \hat{V}_{d,t}^s \) remains quasi-zero. Therefore, aggregate output almost perfectly follows the responses of the wage inflation
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effects, \( \frac{\phi^b_w}{(\varepsilon^b_w - 1)N^b} (\tilde{\pi}_{w,t} - \beta E_t \tilde{\pi}_{w,t+1}) \) and \( \frac{\phi^s_w}{(\varepsilon^s_w - 1)N^s} (\tilde{\pi}_{w,t} - \gamma E_t \tilde{\pi}_{w,t+1}) \), and the shadow value of durables for the borrowers \( \tilde{V}^b_{d,t} \) when monetary tightening occurs. Even though \( \tilde{V}^b_{d,t} \) responds positively, the wage inflation effects respond negatively at a greater magnitude. As a result, aggregate output \( \tilde{Y}_t \) responds negatively.

As shown in <Figure 1>, when monetary tightening takes place, collateral constraints enable wage inflation to respond more negatively. Therefore, the wage inflation effect in the model with collateral constraints is more powerful than that in the model without collateral constraints \( (\omega = 0) \). Consequently, collateral constraints accelerate aggregate output in the model adding sticky nominal wages after monetary tightening.

Next, aggregate nondurable consumption \( \hat{C}_t \) can be similarly expressed as a function of \( \hat{q}_t \), \( \tilde{V}^b_{d,t} \) and \( \tilde{V}^a_{d,t} \), using the equations (47) and (48), that is, \( \hat{V}^b_{d,t} = \hat{q}_t + \hat{U}^b_{c,t} = \hat{q}_t - \hat{C}^b_t \), \( \hat{V}^a_{d,t} = \hat{q}_t + \hat{U}^a_{c,t} = \hat{q}_t - \hat{C}^a_t \):

\[
\hat{C}_t = \frac{\omega C^b}{\omega C^b + (1 - \omega) C^a} \hat{C}^b_t + \frac{(1 - \omega) C^a}{\omega C^b + (1 - \omega) C^a} \hat{C}^a_t \\
= \frac{\omega C^b}{\omega C^b + (1 - \omega) C^a} (\hat{q}_t - \tilde{V}^b_{d,t}) + \frac{(1 - \omega) C^a}{\omega C^b + (1 - \omega) C^a} (\hat{q}_t - \tilde{V}^a_{d,t}) \tag{50}
\]

After monetary tightening, the relative durable price \( \hat{q}_t \) responds negatively. Savers reduce their nondurable purchases proportionally. This is because the shock hardly affects the utility they derive from owning extra durables, as \( \tilde{V}^a_{d,t} \) is quasi-zero. However, to borrowers, there are reasons to substitute away from nondurables towards durables, which is reflected by a positivity in \( \tilde{V}^b_{d,t} \). Given that the response of the relative durable price \( \hat{q}_t \) is almost the same regardless of whether the models include collateral constraints, aggregate
nondurable consumption $\hat{C}_t$ must respond more negatively in the model with collateral constraints.

As a result, aggregate durable spending $\hat{I}_t$ must respond more negatively in the model with collateral constraints. This is because first, the response of aggregate output responds more negatively than that of nondurable consumption and second, the gap of the responses between aggregate output and aggregate nondurable consumption in the model with collateral constraints is larger than in the model without collateral constraints. That is, collateral constraints accelerate the procyclical response of durable spending in the sticky nominal wage model.

2. Flexible Nominal Wages and Collateral Constraints

As Sterk (2010) explains in the general equilibrium setting, collateral constraints decelerate the response of durable spending to monetary policy. Specifically, when borrowing by the borrowers decreases as a consequence of tighter collateral constraints in response to monetary tightening, it is required that the savers save less to remain in equilibrium. The savers then buy durable goods as an alternative way of saving. This kind of behavior weakens intuition that collateral constraints help generate the negative response of durable spending by causing a decrease in durable purchases by the borrowers. Moreover, although the shadow value of durables for savers is quasi-constant, the shadow value of durables for borrowers increases after a monetary contraction because durable goods work as collateral for borrowing and their borrowing constraints become tighter. Subsequently, the borrowers weaken the effect of the constrained ability to borrow on their durables purchases by substituting away from nondurables and working more. This makes aggregate output increase. Moreover, the borrowers have strong incentives to substitute away from nondurables and buy durables instead. This leads to a larger decrease of aggregate nondurable consumption, and aggregate
durable spending increases more greatly following the increase in aggregate output.

As the assumption of flexible nominal wages is one special case of the sticky nominal wages model above with $\phi_{w}^{b} = \phi_{w}^{s} = 0$, the log-linear approximation of aggregate output $\hat{Y}_{t}$ can be expressed as the following reduced equation:

$$\hat{Y}_{t} = \frac{Y_{c}}{Y_{c} + Y_{d}} \hat{Y}_{c,t} + \frac{Y_{d}}{Y_{c} + Y_{d}} \hat{Y}_{d,t}$$

$$= \frac{\omega N^{b}}{\omega N^{b} + (1 - \omega) N^{s}} \hat{N}_{t}^{b} + \frac{(1 - \omega) N^{s}}{\omega N^{b} + (1 - \omega) N^{s}} \hat{N}_{t}^{s}$$

$$= \frac{\omega N^{b}}{\omega N^{b} + (1 - \omega) N^{s}} \frac{1}{\phi^{b}} (\hat{V}_{d,t}^{b} + \hat{w}_{t} - \hat{q}_{t})$$

$$+ \frac{(1 - \omega) N^{s}}{\omega N^{b} + (1 - \omega) N^{s}} \frac{1}{\phi^{s}} (\hat{V}_{d,t}^{s} + \hat{w}_{t} - \hat{q}_{t})$$

(51)

Aggregate output remains nearly constant in the model without collateral constraints and increases in the model with collateral constraints after monetary tightening. To be specific, the real wage in units of durables $\hat{w}_{t} - \hat{q}_{t}$ is 0. Given that $\hat{V}_{d,t}^{s}$ remains quasi-zero, aggregate output almost perfectly follows the positivity in the shadow value of the borrowers $\hat{V}_{d,t}^{b}$ when monetary tightening takes place.

Since the response of nondurable consumption $\hat{C}_{t}$ to monetary tightening is analogous in the sticky nominal wages model, it responds more negatively when households face collateral constraints.

There is a clear reason why collateral constraints lead to a more positive response of aggregate durable spending. When the response of aggregate output is more positive after adding collateral constraints, and nondurable consumption falls more greatly, then the aggregate resource constraint can be
satisfied only if production in the durable goods sector responds more positively. Consequently, collateral constraints decelerate the response of durable spending in the flexible nominal wage model.

V. Simulation Results

If the prices of durable goods are sticky, we can numerically investigate that the response of a two-sector sticky price model with collateral constraints to monetary policy shocks.

Collateral constraints accelerate durable spending to monetary shocks in the model with sticky nominal wages regardless of durables price stickiness. Figure 2 plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable spending, and aggregate output to monetary tightening in the sticky nominal wages model. Aggregate nondurable consumption $\hat{C}_t$ responds more negatively in the model with collateral constraints after monetary contraction. Also, aggregate durable spending $\hat{I}_t$ must respond more negatively. This is because first, the response of aggregate output responds more negatively than that of nondurable consumption and second, the gap of the responses between aggregate output and aggregate nondurable consumption in the model with collateral constraints is larger than in the model without collateral constraints. That is, collateral constraints amplify the procyclical response of durable spending.

On the other hand, as Sterk (2010) shows, durable spending is decelerated by collateral constraints to monetary shocks in the model with flexible nominal wages regardless of the price stickiness of durable goods. Figure 3 plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable spending, and aggregate output to monetary tightening in the flexible nominal wages model. When the response of aggregate output is more positive after adding collateral constraints, and nondurable consumption falls
more greatly, then the aggregate resource constraint can be satisfied only if production in the durable goods producing sector responds more positively. Consequently, augmenting the model with collateral constraints decelerates the response of durable spending.

VI. Adding the Adjustment Costs in Durable Investment

Although the model with sticky nominal wages generates the negative response of durable spending in the initial period, the sensitivity of durable goods to a monetary contraction is too much larger than that of nondurables, which is at odds with empirical data. As Carlstrom and Fuerst (2010) shows, the assumption of sticky nominal wages leads to the rapid over-shooting of durable spending. Thus, they add the adjustment costs in durable investment, i.e. durable spending, to resolve this problem. This paper considers the model with sticky nominal wages and adjustment costs in durable investment.

Durable investment increases the households' durables stock according to:

\[
D_t^b = F(I_t^b, I_{t-1}^b) + (1 - \delta)D_{t-1}^b
\]

\[
D_t^s = F(I_t^s, I_{t-1}^s) + (1 - \delta)D_{t-1}^s
\]

where durable investment adjustment costs are given by

\[
F(I_t^b, I_{t-1}^b) = \left(1 - S\left(\frac{I_t^b}{I_{t-1}^b}\right)\right)I_t^b
\]

\[
F(I_t^s, I_{t-1}^s) = \left(1 - S\left(\frac{I_t^s}{I_{t-1}^s}\right)\right)I_t^s
\]

The function \(S(\cdot)\) represents adjustment costs that are incurred when the level of durable investment changes over time. This research assumes that
$S(1) = 0$, $S'(1) = 0$, so that there are no adjustment costs in the steady state. $\kappa \equiv S''(1) > 0$ is set in such a way that the volatility of durable spending to monetary shocks is about four times larger than that of nondurable consumption. In this model, equations (52) and (53) replace equations (6) and (15) respectively.

<Figure 4> plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable spending, and aggregate output to monetary tightening in the sticky nominal wages model with durable investment adjustment costs. The model with adjustment costs in durable investment resolves the over-shooting problem. Also, like the previous model, total output decreases more greatly and both nondurable consumption and durable spending respond more negatively after monetary tightening than in the model without collateral constraints.

**VII. Conclusions**

Our paper shows that collateral constraints on the household's side can play a critical role in a two-sector sticky price model. While many papers have stressed the role of credit frictions in a one-sector sticky price model, this has been little explored in the setting of the two-sector sticky price model. Although Iacoviello (2005) and Iacoviello and Neri (2010) study the general equilibrium model with collateral constraints, they do not focus on how it affects spending on durable goods. Monacelli (2009) emphasizes the role of collateral constraints in accelerating the response of durable spending in response to monetary policy shocks, but Sterk (2010) argues otherwise, saying that collateral constraints decelerate the response of durable spending to monetary shocks in the model with flexible nominal wages. However, our paper shows that collateral constraints accelerate the effects of monetary policy shocks on durable spending in the presence of sticky nominal wages. The main reason is that collateral constraints strengthen the wage inflation effect,
having a significant effect on labor allocation. Many research papers such as Kim (2000), Huang and Liu (2002), Christiano et al (2005), and DiCecio (2009) have proved that nominal wage rigidity captures successfully some key features of business cycles. Hence, it is legitimate to say that the role of collateral constraints is important in the two-sector sticky price model as well as in the one-sector sticky price model.
〈Figure 1〉 Responses of Wage Inflation to the Monetary Policy Shocks in the Sticky Nominal Wages Model

Note: Responses to monetary tightening (25 basis point innovation) in the sticky nominal wages model without and with collateral constraints. The responses are plotted as deviation from steady state in percentage point.
(Figure 2) Responses to the Monetary Policy Shocks in the Sticky Nominal Wages Model

Note: Responses to monetary tightening (25 basis point innovation) in the sticky nominal wages model without and with collateral constraints. Response of nominal interest rate is plotted as deviation from steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.
Note: Responses to monetary tightening (25 basis point innovation) in the flexible nominal wages model without and with collateral constraints. Response of nominal interest rate is plotted as deviation from steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.
Figure 4: Responses to the Monetary Policy Shocks in the Sticky Nominal Wages Model adding Durable Investment Adjustment Costs

Note: Responses to monetary tightening (25 basis point innovation) in the sticky nominal wages model adding durable investment adjustment costs without and with collateral constraints. Response of nominal interest rate is plotted as deviation from steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.
References


