We study the inter-sectoral consequences of a general disequilibrium in a global production network in which each sector’s output is used as an input for others. We find that the presence of excess demand in any single sector implies positive profits for all others that use the constrained good as an input. As a corollary, in a general disequilibrium in which more than one market is stuck with excess demand, with other sectors being clear in this respect, everyone enjoys positive profits. By implication, the paradoxical results derived from a model of interdependence associate a higher chance of excess demand with an overall rise in profit levels during booms, thereby reprising the Keynesian sense of effective demand-driven business cycles.

**Keywords:** Input-output matrix, Profits, Interdependence, Disequilibrium, Business cycles

**JEL Classification:** C67, D57, E12

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* This paper was supported by research funds from Chonbuk National University in 2015.

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I. Introduction

Economists have long assumed that every market clears continuously. However, everyday observation contradicts the assumption. According to (the contraposition of) the Walras’ Law, the presence of one single market in excess demand (or in excess supply) suffices to show that the whole economy is in a disequilibrium state. Example: A fellow who takes a bus this morning and finds no seats available (or sees many seats vacant) can assure his friends that the world economy is currently not in a Walrasian equilibrium. A prerequisite is that his friends have taken EC2XXX, Intermediate Microeconomics. – His friends, we hope, would tell in reply more about their own experiences (perhaps more intriguing) in markets for automobiles, houses, loans, telecom, internet, and so on.

It is this article’s main premise that a Walrasian tatonnement process may fail: In a real world, the market prices vector may contain some mispriced entries for various reasons that are beyond individual agent’s control. The premise itself is not new [see Benassy(2008) for a review]. But the main aim of the present study differs from the literature. Stepping aside specific market frictions and reasons behind the failure of a Walrasian tatonnement process, we are interested in inter-sectoral consequences of a general disequilibrium in a roundabout production economy.

To the aim, we consider a simple many-sector production economy where each sector’s output is used as inputs for other sectors. This input-output structure is a realistic shot of modern industrialized economy: According to the “Input-Output Use Tables” from the Bureau of Economic Analysis (BEA), there are found few zeros at the detailed level of industries, which suggest strong interdependence among sectors in the U.S. economy. We combine the input-output structure with our everyday observation of market-clearing failure in automobiles, credit, transportation, etc., and ask what we can infer about the other parties’ profits from the presence of excess demand in some markets.

Our analysis shows that, if one market is stuck in excess demand state,
firms who use the constrained good as input make positive profits at their partial
equilibrium. As a corollary, in a general disequilibrium at which more than one
market is stuck in excess demand while other markets clear, everyone enjoys
positive profits. By implication, the paradoxical result implies a tight association
between higher chance of excess demand and overall rise in profit level during
booms, whereby the Keynesian sense of effective demand-driven business cycles
revives from our microeconomic analysis of general disequilibrium.

II. The Economy

1. The Input-Output Matrix

Consider a production economy that consists of a continuum of sectors,
\( J = \{ j | j \in [0,1] \} \). In turn, each sector \( j \) consists of a continuum of identical
firms of measure one, which produce a homogeneous good \( j \) and make symmetric decisions. So the world is visualized as the unit rectangle, \([0,1]^2\),
with a continuum of firms upon the space.

Markets are interconnected one another through input-output process by
which each sector’s output is used as inputs for other sectors. For the virtue
of tractability, we assume that production technology for all firms is
homogeneous of degree one. Specifically, the economy’s input-output process
is modelled in the line with the Dixit-Stiglitz[Dixit and Stiglitz(1977)].

Consider a sector \( j \in J \), and pick up an arbitrary firm from the sector \( j \).
One can think of the firm as a representative “stand-in” in the sector due to
the homogeneous production function of degree one and a continuum of
measure one. Let \( Y_j \) denote output \( j \) produced by the representative firm
having access to the following technology:

\[
Y_j = \left[ \int_{k \in J} \left( X_{j,k} \right)^{\eta} \left( Z_{j,k} \right)^{\frac{n-1}{\eta}} \; dk \right]^{\frac{1}{\eta}}, \; 1 < \eta < \infty,
\]
where $X_{j,k}$ measures the technical contribution of $k$th good to production of $j$th good, and thus can be seen as a quality-dimension of $k$th good from the viewpoint of $j$th sector. $Z_{j,k}$ is the input amount of $k$th good used by the representative firm in the $j$th sector. Inputs are neither perfect substitutes nor perfect complements one another; $1 < \eta < \infty$. Also, the input-output network is assumed to work simultaneously across sectors, and therefore in principle the firm can use its own output as input for output. If this is the case, there are some sector $j$’s showing $X_{j,j} \neq 0$.

By matching every two sectors out of $J$ and collecting their mutual technical contributions, we construct a $J \times J$-continuum matrix, $X = [X_{j,k}]_{(j,k) \in J \times J}$. As a whole, the matrix $X$ defines the fundamental interdependence for this production economy and nests all cases of vertical in-line production relation. For example, if $(j,k)$th entry equals zero but $(k,j)$th entry is not (i.e., $X_{j,k} = 0$, $X_{k,j} \neq 0$), then the two sectors $j$ and $k$ are in a vertical production relation in the sense that we do not need good $k$ to produce good $j$ but need $j$ to produce $k$.

**Assumption 1.** (i) $X_{j,k} \geq 0$ for every pair of $(j,k) \in J \times J$. (ii) Let $X_{j,J}$ be a vector from a collection of the $J$-continuum of the entries in th row of the matrix $X$; and $X_{J,k}$ be a vector from a collection of the $J$-continuum of the entries in kth column of $X$. It is assumed that $x_{j,J} \neq 0^J$ and $x_{J,k} \neq 0^J$, i.e., every good needs at least one input and every good is useful for at least one sector. (iii) All the entries in the vector $x_{J,k}$ sum to one; i.e.,

$$\int_{j \in J} X_{j,k} \, dj = 1.$$

The first part of the assumption is made because when $X_{j,k} < 0$, $j$th sector does not use $k$ good and in general when there is an adverse mutual relation between some sectors, the bilateral trade will be voluntarily removed. We need the second part of the assumption just to eliminate trivial trading equilibria, for
example, one or more equilibria forced when $X$ is a null matrix, which should be out of interest for most cases. The third part of the assumption is for normalization of the relative contribution of $k$ to many user sectors.

2. Input Demand and Output Use

Now consider a sector $k \in J$. Let $Z_k$ denote the economy’s total use of the $k$th good: that is,

$$Z_k = \int_{j \in J} Z_{j,k} \, dj.$$

Let also $p$ denote a $J$-continuum vector of market prices ($p = [P_j]_{j \in J}$). At any given prices vector $p$, the economy-wide use of good $k$, $Z_k$, cannot exceed the total production of good $k$, $Y_k$. That is, $Z_k \leq Y_k$ for each market $k \in J$. In what follows, we elaborate this constraint for each sector $k$ in terms of aggregate excess demand and actual total use in production, generalize them across all the sectors in $J$, and use to define partial and general equilibrium, and general disequilibrium.

Let $Z_{j,k}^d$ denote the demand for good $k$ desired by the sector, and $Z_k^d$ the aggregate demand for good $k$: that is,

$$Z_k^d = \int_{j \in J} Z_{j,k}^d \, dj.$$

For the technical interdependence matrix $X$ given, the aggregate demand function of good $k$ is a real-valued function of $p$: that is, $Z_k^d(p; X)$. Similarly, we have the economy’s total use of good $k$ and its total production as a mapping from the joint space of $p$ and $X$ into $\mathbb{R}_+$, respectively: that is, $Z_k(p; X)$ and $Y_k(p; X)$.

**Definition 1.** Let $z_k^d$ denote the aggregate excess “demand” for good $k$. For the technical interdependence matrix $X$ given, the aggregate demand function for $k$ is a real-valued function of $p$. 

\[ z^d_k(p; X) = Z^d_k(p; X) - Y^d_k(p; X). \]

When \( z^d_k(p; X) > 0 \), the economy-wide demand for good \( k \) exceeds its total production and thus there is excess demand for good \( k \). When \( z^d_k(p; X) < 0 \), there is excess supply of good \( k \).

**Definition 2.** Let \( z_k \) denote the economy’s total excess “use” of good \( k \). For \( X \) given, the aggregate excess use function for \( k \) is a real-valued function of \( p \),

\[ z_k(p; X) = Z_k(p; X) - Y^d_k(p; X), \]

*bounded* above by zero.

Note the relationship between the aggregate excess demand function \( z^d_k(p; X) \) and the aggregate excess use function \( z_k(p; X) \). (i) \( z_k(p; X) = 0 \) is implied by either \( z^d_k(p; X) > 0 \) or \( z^d_k(p; X) = 0 \): that is, we find \( z_k(p; X) = 0 \) when the aggregate demand for good \( k \) does either equal its total production or exceed. (ii) \( z_k(p; X) < 0 \) is observed if and only if \( z^d_k(p; X) < 0 \). Definition 2 refines the constraint \( Z_k \leq Y^d_k \) for each sector \( k \) by the boundedness of the aggregate excess use function for each \( k \), \( z_k(p; X) \leq 0 \).

**Definition 3.** Let \( z^d \) denote the aggregate excess “demand” for all varieties. The aggregate excess demand function for the set \( J \) is a vector-valued function,

\[ z^d(p; X) = [z^d_k(p; X)]_{k \in J}. \]

Let \( z \) denote the aggregate excess “use” for all varieties. The aggregate excess use function for the set \( J \) is a vector-valued function,

\[ z(p; X) = [z_k(p; X)]_{k \in J} \]

*bounded* above by \( 0^J \), where \( 0^J \) denotes a \( J \)-continuum null vector.
Like at the level of each single market, \( z(p; X) = 0^J \) is implied by either \( z^d(p; X) > 0^J \) or \( z^d(p; X) = 0^J \), and \( z(p; X) < 0^J \) if and only if \( z^d(p; X) < 0^J \). Using this definition, we state the economy-wide resource constraints across sectors as follows:

\[
z(p; X) \leq 0^J
\]  

(2)

at any given prices vector \( p \) and for the technical interdependence matrix \( X \) given.

In the next section, we close our model economy by defining its equilibrium and disequilibrium but without explicit introduction of household’s sector which supplies labor as a unique form of input and consumes a basket of final goods. However, one can easily extend it by taking the whole block of our simple production network to a typical model economy in which markets for final consumption and labor are explicitly presented and both assumed to clear. In this paper, we leave out such a tedious and typical extension for the virtue of simplicity. Intact are our results below on firm profits as an inter-sectoral consequence of goods market failure where production is interdependent.

### III. Profits in Equilibrium and Disequilibrium

Each individual firm takes a prevailing market prices vector \( p \) as given. Henceforth, without loss of generality, we draw attention to a particular section of input-output network in this multi-sector roundabout production economy, and focus on a representative “stand-in” doing a business in sector \( j \in J \), of which output market is currently in partial equilibrium (as defined below) at a prevailing market prices vector \( p \). We will shorthand the representative firm by “rep”.


Definition 4. (i) A market $k$ is said to be in a partial equilibrium at a prevailing prices vector $p$, if $z^d_k(p; X) = 0$ for the technical interdependence matrix $X$ given. (ii) The economy is said to be in a general equilibrium at a prevailing prices vector $p$, if $z^d(p; X) = 0$. (iii) The economy is said to be in a general disequilibrium, if at least one market $k$ is out of a partial equilibrium at a prevailing prices vector $p$, $z^d_k(p; X) \neq 0$. Among many possible states of general disequilibrium, the economy is said to be in an input-constrained disequilibrium, if at least one market $k$ faces excess demand at $p$, $z^d_k(p; X) > 0$.

1. General Equilibrium

Suppose the economy is known to be currently in $z^d(p; X) = 0$ and thus $z(p; X) = 0$. It is assumed that the rep maximizes its profits by choosing optimal quantities of inputs across sectors in $J$, $[Z_{j,k}]_{k \in J}$:

$$\max_{[z_{j,k}]_{k \in J}} \pi_j = P_j Y_j - \int_{k \in J} P_k Z_{j,k} dk;$$

subject to the production technology (1) and taking $p$ and $X$ as given. We find the first-order conditions for the rep’s profit maximization problem as follows:

$$P_j = M_j$$

$$Z_{j,k} = X_{j,k} Y_j \left( \frac{P_k}{M_j} \right)^{-\eta}$$

for every input $k \in J$. Here $M_j$ is the Lagrangian multiplier associated with the production technology (1), and thus marginal cost of $j$th good production. Equation (4) is the standard equalization condition between marginal cost and
price. Equation (5) is the rep’s optimal demand schedule for each input \( k \).
Since the inputs are imperfect substitutes one another \( (1 < \eta < \infty) \), the rep
demands a positive amount of \( k \)th input \( (Z_{j,k} > 0) \) if \( X_{j,k} > 0 \). Reading the
equations is straightforward: the rep \( j \) demands input \( k \) more when \( k \) delivers
higher productivity \( (X_{j,k}) \), when it produces more output \( (Y_j) \), and when
input \( k \)’s price is relatively cheaper to its own price \( (P_k/P_j) \).

By substituting back the optimal input use of good \( k \) (5) into the production
function (1), one can show that

\[
M_j = \left[ \int_{k \in J} X_{j,k}(P_k)^{1-\eta}dk \right]^{1-\eta}. \tag{6}
\]

Intuitively, the marginal cost of production, \( M_j \), will increase in input prices
and decrease in the technical contributions of inputs in use. Let \( C_j \) denote the
average cost of good \( j \) production given by

\[
C_j = \frac{\int_{k \in J} P_k Z_{j,k} dk}{Y_j}. \tag{7}
\]

The rep makes optimal production decision where marginal cost of production
equals average cost.

**Lemma 1.** It holds that \( C_j = M_j \) at the rep’s optimal decision.

*Proof.* See Appendix.

**Proposition 1.** There exists at least one general equilibrium with positive
production and trade, and zero profits.

*Proof.* See Appendix. Essentially, the proof for positive production and trade
follows the same line of the Welfare Theorem. It shows that a central planner
who directs everyone’s output (producible resources) according to how much it can contribute to others’ productivity leads the economy to the same outcome a competitive general equilibrium attains. And the proof for zero profits utilizes the fact that in such a general equilibrium, there is a prices vector \( p \) such that \( z^d(p; X) = 0 \) at which every rep meets the first-order condition (4) and Lemma 1.

By contraposition, if one sees some reps attain positive profits at \( p \), this tells us that the roundabout production economy as a whole is in a disequilibrium at \( p \).

2. General Disequilibrium

Now consider an input-constrained general disequilibrium in which some markets are in \( z^d_k(p; X) > 0 \), some markets in \( z^d_k(p; X) = 0 \), and the rest markets in \( z^d_k(p; X) < 0 \). Let us focus on the reps whose markets are in their partial equilibrium, \( z^d_k(p; X) = 0 \), while the economy as a whole being found at a disequilibrium state.

Then each individual rep should take into account the state of the economy during its optimal production decision at the prevailing market prices vector \( p \) for given \( X \). The rep is aware of the economy-wide resource constraints across sectors, (2). If \( z^d_k(p; X) > 0 \) for some \( k \)'s that the rep wants to use as inputs, the rep will face some rationing of \( k \)'s among all the other sectors who want to use. For the virtue of simple exposition, we introduce a rationing rule that allows us to keep the representative framework throughout.

Assumption 2. If \( z^d_k(p; X) > 0 \) for some \( k \), each sector \( j \) who demands th input has access to the market \( k \) according to \( Z_{j,k} = X_{j,k}Y_{k,i} \), i.e., how productively the sector \( j \) can use the scarce resource the sector \( k \)'s output.
When the aggregate demand exceeds the total production of good $k$, the economy-wide use of good $k$ is binding and thus the aggregate excess use function comes in effect, $z_k(p; X) = 0$. This is when some goods are more or less mispriced and therefore the prevailing market prices vector $p$ departs from a Walrasian. The rationing rule made in Assumption 2 is one possible allocation rule that is known “efficient” by Lemma 1 in the sense that it will have been actually at work if the market prices vector $p$ equals a Walrasian $p^*$. Nevertheless, the rationing rule is also quite general, though it sounds restrictive. Notice that we have not imposed any further restrictions on the technical efficiency matrix $X$ other than the Assumption 1. So even some discriminative rationing rules (e.g., by which some particular sectors have larger access to some specific goods) can be easily presented in similar terms of the rule by having the matrix $X$ consist of technical part and bureaucratic preferences. As this kind of generalized rationing rules raises the number of terms and notations without analytic benefits, we hire the Assumption 2 within a representative framework.

The rep $j$ who maximizes its profits (3) takes into account the disequilibrium state of the economy in terms of the economy-wide resource constraint (2) at a given prices vector $p$. And with measure one for each sector, the economy-wide resource constraint will be read by each firm in th sector as

$$z(j, p; X) \leq 0^J,$$

(8)

where $z(j, p; X)$ is a $J$-continuum vector as a collection of a real-valued function $z_k(j, p; X)$ defined by

$$z_k(j, p; X) = Z_{j,k}(p; X) - X_{j,k} Y_k(p; X)$$

(9)

for every $k$: that is,

$$z(j, p; X) = \left[z_k(j, p; X)\right]_{k \in J}$$
Notice that under Assumption 1, it holds for each $k$ that
\[ \int_{j \in J} z_k(j, p; X) \, dj = z_k(p; X) \]
in its relation to the aggregate excess use function, $z_k(p; X)$.

Having all at one place, the rep $j$ maximizes its profits (3) subject to both the technological constraint (1) and the resource constraint (8). The first-order conditions include the price-marginal cost equalization (4) as standard, and

\[
Z_{j,k} = X_{j,k} Y_j \left( \frac{P_k + \gamma_{j,k}}{M_j} \right)^{-\frac{1}{\eta}},
\]

(10)

\[
\gamma_{j,k} z_k(j, p; X) = 0, \quad \gamma_{j,k} \geq 0,
\]

(11)

for every $k \in J$. Here $M_j$ is the marginal production cost of good $j$ as defined before, and $\gamma_{j,k}$ is the Lagrangian multiplier associated with the resource constraint (8) for each entry $k$ in the vector. The usual interpretation of shadow costs applies here: $\gamma_{j,k}$ is the shadow price of binding the constraint (8) for each $k$. If $z_k^d(p; X) \leq 0$, $\gamma_{j,k} = 0$. Otherwise, $\gamma_{j,k} > 0$.

Equation (10) is the rep’s effective optimal demand schedule for $k$th input. One can think of the sum of market price plus shadow price, $P_k + \gamma_{j,k}$, as effective cost of $k$th input. And using (4), we know $(P_k + \gamma_{j,k})/M_j = (P_k + \gamma_{j,k})/P_j$: i.e., the effective input cost to output price. So it is clear to see what (10) says: the rep demands $k$th input more when it finds the contribution of th input to its production of output $j$ and demands less when the effective cost of input, $(P_k + \gamma_{j,k})/P_j$, is higher. In general, (10) nests (5) as a special case, which holds only when the state of economy is pre-specified, for example, to be in a general equilibrium, and thus when equation (11) does not come into effect.

We develop a tractable metric for sectoral disequilibria using the notion of effective costs and prices that can be seen as a generalization of marginal and average production costs known in the previous section of general equilibrium.

By substituting (10) back into (1), we have now marginal production costs $M_j$...
at optimum as a function of input prices and the technical contributions of inputs:

\[
M_j = \left[ \int_{k \in J} X_{j,k}(P_k + \gamma_{j,k})^{1-\eta} dk \right]^{1-\eta}. \tag{12}
\]

As before, the marginal cost of production, \( M_j \), increases in input prices and decreases in technical contributions of inputs in use. In addition, it increases now in the shadow costs of inputs. Average production cost \( C_j \) is defined as before, by (7). Substituting (10) into (7), now we have

\[
C_j = \frac{\int_{k \in J} P_k Z_{j,k} dk}{Y_j} = \int_{k \in J} P_k X_{j,k} \left( \frac{P_k + \gamma_{j,k}}{M_j} \right)^{-\eta} dk = (M_j)^{\eta} \left[ \int_{k \in J} P_k X_{j,k} (P_k + \gamma_{j,k})^{-\eta} dk \right] = (M_j)^{\eta} (\overline{M}_j)^{1-\eta},
\]

where \( (\overline{M}_j) = \left[ \int_{k \in J} X_{j,k} (P_k + \gamma_{j,k})^{-\eta} dk \right]^{1-\eta} \). In brief, at the rep’s optimal decision, the average production cost is given by

\[
C_j = (M_j)^{\eta} (\overline{M}_j)^{1-\eta}. \tag{13}
\]

Notice that \( \overline{M}_j \) is obtained when the integrand of \( M_j \) is multiplied by \( P_k (P_k + \gamma_{j,k}) \). So it is clear that the distance between \( M_j \) and \( \overline{M}_j \) (and therefore between \( M_j \) and \( C_j \)) is fully governed by the relative size of the prevailing market prices over the shadow costs. This means that their distance can be taken as a useful metric for sectoral disequilibria from the viewpoint of the rep \( j \).
Lemma 2. It holds that $C_j \leq M_j$ at the rep's optimal decision for any given $(p, X)$.

Proof. See Appendix.

Notice that $M_j = \bar{M}_j$ only when $\gamma_{j,k} = 0$ for every input $k$, i.e., only when $z_k^d(p; X) \leq 0$. If this is the case, the average production cost equals to the marginal production costs, $C_j = M_j$. In other words, Lemma 2 generalizes Lemma 1. If the economy is in a state of an input-constrained disequilibrium, we find that the above result holds with strict equality: $M_j < \bar{M}_j$ and $C_j < M_j$.

Proposition 2. A rep makes positive profits if and only if it cannot buy at least one input as many as it wants.

Proof. ["if" part] Consider a rep $j$ faces that $\gamma_{j,k} > 0$ for at least one $k \in J$. Under the state of the economy, the first-order condition (4) and Lemma 2 jointly tell us that $C_j < M_j = P_j$. Consequently, $\pi_j = P_j Y_j - C_j Y_j > 0$ if $\gamma_{j,k} > 0$ for at least one $k \in J$. ["only if" part] Suppose the contrary: that is, if $\gamma_{j,k} = 0$ for all $k$’s, the rep $j$ makes positive profits. However, the rep having $\gamma_{j,k} = 0$ for all $k$’s will make optimal decision at $C_j = M_j = P_j$, which leads to $\pi_j = 0$. So the supposition contradicts.

By contraposition of Proposition 1, we already know that if we observe some firms attain positive profits at $p$, the economy as a whole must be in a state of general disequilibrium. Proposition 2 refines the implication of the previous result and tells us that such a general disequilibrium is an input-constrained disequilibrium, where by definition, at least one market fails to meet demand. However, this inter-sectoral consequence of a general disequilibrium is not entirely obvious and even sounds paradoxical—how one can make positive profits whenever it cannot buy as many as he wants at prevailing market prices?
3. A Paradox?

The key mechanism behind the inter-sectoral consequence is related to the behaviour of marginal costs relative to average costs, as an optimal reflection of the state of the economy into individual workplace. If a rep cannot buy some inputs as many as it wants at \( p \), it will substitute them with other inputs. However, because the inputs are imperfect substitutes one another, the rep will have to bear some efficiency loss when producing its output. The shadow costs vector, \( [\gamma_{j,k}]_{k \in J} \), captures the efficiency loss that the rep has to bear in using the inputs. As the rep makes rational decisions at the margin, it will take the size of the efficiency loss into account of its marginal production cost. Moreover, since the rep equates the marginal cost with the market price of its output at its optimal production, the market price will reflect the economic value of the efficiency loss. Intuitively, the wider range of inputs is constrained and the more severely constrained, the marginal cost will rise. This is what equation (12) tells us.

Nevertheless, when it comes to average costs, the shadow costs become a double-edged sword.\(^1\) On the other hand, the shadow costs lower the average cost, because the shadow costs are mostly associated with excessive use of relatively “underpriced” inputs for given \((p; X)\). Equation (13) concisely captures the two contrasting roles of the shadow costs. Clearly, \((\tilde{M}_j)^\eta\), represents “efficiency loss” in production and rises as the shadow costs become larger; whereas \((\tilde{M}_j)^{1-\eta}\) represents overuse of “underpriced” inputs and thus decreases in the shadow costs. Facing the two opposite forces at work, the rep will see the marginal cost exceeds the average cost and find its business profitable unless the shadow costs vector is a null at the prevailing market prices vector.

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1) In principle, the average cost itself does not condition one’s optimal production decision, and instead counts on market prices and actual purchases of inputs. However, since the average cost at optimum can be also expressed in terms of the shadow costs and market prices, we can contrast the average cost with the marginal cost w.r.t. their identical arguments. In this respect we discuss about the role of the shadow costs in carving the average cost at one’s optimal production decision.
As a corollary, it immediately follows that if at least two markets are stuck in excess demand while every market is interconnected in terms of the matrix $X$, then every firm in this production economy could make positive profits. Of course, this is where no markets are currently in excess supply just as in a “repressed inflation” regime [Green and Laffont (1981)]. Such situations cannot be ruled out because with a non-Walrasian market prices vector, the presence of excess demand in one market does not necessarily imply the presence of excess supply in other markets.

**Ⅳ. Discussion**

Proposition 1 holds when no entries are mispriced. Proposition 2 holds when some entries are mispriced downward. Neither situations underlying the two results can be ruled out. We discuss two key aspects of the model environment underlying these results.

**Perfect Competition and Zero Profit Condition:** One may wonder if the results hold here because the free-entry argument was inactivated. However, when it comes to a pure system of logic, the results come in full of scenes of the free-entry argument. In the model, each sector contains infinitely many number of firms who are free to move across sectors with free access to the production technology of homogenous of degree one, and always equate their marginal costs with given market prices. Moreover, where at least one market is stuck in excess demand, the free-entry argument does not work as a forcing device toward zero-profit due to the inter-sectoral input-output network of this production economy: For example, when positive profits attract more firms, it will only aggravate the current state of sectoral disequilibria because newcomers mean more demand for the inputs already in excess demand. To explicitly incorporate the inter-sectoral relocation within a simple static setup, one might introduce a non-uniform measure of sectors and let each sector be of different
size in general, whereby the free-entry argument is brought up at a first place in a form of perfect profit equalization across sectors rather than as a forcing device toward zero-profit. But still then, one would still have to see the first-order conditions remain intact irrespective of sector’s scale, and so do both results above.

Such an extension that incorporates the inter-sectoral relocation would be fruitful if it is made in a full dynamic setup. However it will be necessary to introduce some justifiers like uncertainty, expectations, inventory transition process, and so on, and by dosing so to rely on a richer solution concept by which economic agents form rational beliefs about all possible states of general disequilibrium and make rational decisions [Gordon(1981)]. For example, Benassy (1986) facilitates the concept of a Nash equilibrium between monopolistically competitive agents.

**Input-Output Structure and Intermarket Relationships:** One may doubt if the assumption of the input-output structure in this note overstates the technological interdependence actually present in our economy, and thus if it unduly highlights the intermarket relationships leading to strong implications from a tiny deviation from a Walrasian state. In the theory, however, the results do not depend on the number of zero entries present in the continuum matrix of input-output technical interdependence, $X$, unless “almost all” entries were forced to be zero. In data, moreover, we see strong interdependence among industries as well as among countries: See, for example, the BEA Use Tables and the KLEM dataset constructed by Jorgenson and his collaborators [Jorgenson et al.(1987), Jorgenson (1990), Jorgenson and Stiroh(2000)]. Huang et al.(2004) attribute the change in the cyclical properties of real wage to the increasing sectoral interdependence in the U.S. economy. In examining the cyclical behaviour of productivity, Basu(1996) finds procyclical inter-sector technical efficiency. See also Basu(1995) and Basu(1995), Basu and Fernald(1997).

Our results relate the input-output structure to a Keynesian general disequilibrium. As well understood from the Keynesian theory of income
determination, Keynes was concerned with the intermarket relationships in a system of markets which do not continuously clear, and in particular with the relationships between goods, labor, and money markets when prices, wages, and interest rates are mispriced for some (possibly various) reasons. So the idea of the Keynesian disequilibrium provides a natural platform (or at least a good starting point) for the study of interconnectedness between many different parts of developed production economy. For example, a casual observation that one’s spending becomes another one’s income is the key to the analysis of multiplier effect in the traditional Keynesian literature. A similar observation that one’s mispriced output turns to another one’s costs and benefits motivates the present study. Proposition 2 is one of many possible consequences upon the input-output network which well captures the modern way of production.

V. Concluding Remarks

This paper studies inter-sectoral consequences of a general disequilibrium, born in the contraposition of the Walras’ Law. It considers a many-sector roundabout production economy where each sector’s output is used as input for other sectors and focuses on a particular section of the input-output network within a representative framework. Our study finds that if one market is stuck in excess demand state, firms who use the constrained good as their input make positive profits; and if so are more than one, every firm makes positive profits.

Though the model environment is static, tersely built, and production-oriented, the proposed results on sectoral disequilibria have wide implications in economics. First of all, our microeconomic analysis of general disequilibrium indicates a close connection between higher chance of excess demand and overall rise in profit level during booms, and revives the Keynesian sense of effective demand-driven business cycles.

Our results can be also appreciated in a context of banking and finance: For
example, credit rationing for loanable funds may indicate positive profits for user sectors of rationed loans, as well as for the banking sector as studied in the existing literature. This is because where the Modigliani-Miller theorem breaks down, funds become imperfect substitutes depending on their sources of finance. Admittedly the input-output production function styled as a Dixit-Stiglitz aggregator may be too restrictive to use it for modelling interconnectedness of economic agents in financial markets. Nevertheless, a generalized functional form of input-output network would help theorize and study the interconnectedness of financial markets, and consequences and implications of mispriced securities in a financing-production economy.
Appendix

Proof of Lemma 1

Substituting back the optimal input use of good \( k \) (5) into the definition of the average production cost (7),

\[
C_j = \frac{\int_{k \in J} P_k Z_{j,k} dk}{Y_j} = \int_{k \in J} P_k X_{j,k} \left( \frac{P_k}{M_j} \right)^{-\eta} dk = (M_j)^\eta \left[ \int_{k \in J} X_{j,k} (P_k)^{1-\eta} dk \right] = (M_j)^\eta (M_j)^{1-\eta} = M_j
\]

where the fourth equality uses (6).

Proof of Proposition 1

(1) Proof of Positive Production and Trade Fix \((0, j, k) \in J \times J \times J\), where the first entry means the sector 0 and the pair \((j, k), j \neq 0 \) and \( k \neq 0 \), is arbitrarily selected. Let the sector 0 be a numeraire with normalization \( P_0 = 1 \) and \( Y_0 > 0 \). Let \( \Delta_{j,k} \) be a real-valued function that maps from \( X_{j,k} \) onto \( \mathbb{R}_+ \). Assume \( \Delta_{j,k} \) is the one that satisfies \( Z_{j,k} = \Delta_{j,k} Y_k \), which says that at optimum the rep \( j \) takes a \( \Delta_{j,k} \) fraction of the total production of good \( k \), \( Y_k \), as an input. On the other hand, \( Z_{j,k} \) is given by the optimal demand schedule (5). Having \( \Delta_{j,k} Y_k \) equal to the optimal demand schedule (5) with (4), we obtain

\[
\frac{P_k}{P_j} = \left( \frac{\Delta_{j,k} Y_k}{X_{j,k} Y_j} \right)^{-\frac{1}{\eta}}. \tag{14}
\]
Guess the functional form of $\triangle_{j,k}$ and specifically let $\triangle_{j,k} = \alpha X_{j,k}$ with $\alpha > 0$. $\triangle_{j,k} = 0$ if output $j$ does not require use of input $k$, i.e., $X_{j,k} = 0$. Our guess is based on the property of the production function that is homogeneous of degree one. We will verify our guess by showing the existence of a mapping $\triangle_{j,k}$ which makes every market satisfy its optimality conditions and the technological possibilities (the production function) for given $X$. Under the conjecture, the production function for good $j$ (1) is given by

$$Y_j = \alpha \left[ \int_{k \in J} X_{j,k} \left( \frac{Y_k}{Y_j} \right)^{\frac{\eta-1}{\eta}} \, dk \right]^\frac{\eta}{\eta-1}. \tag{15}$$

At the same time, equation (14), the optimal demand schedule under the conjecture, can be rewritten as

$$\frac{P_k}{P_j} = \left( \frac{Y_k}{Y_j} \right)^{-\frac{1}{\eta}}.$$

Then, by the same token, we can generate the bilateral relation from every pair of two sectors $(j, k) \in J \times J$. Regarding the relation between the numeraire sector 0 and arbitrary $k \neq 0$, we will that $\frac{P_k}{P_0} = \left( \frac{Y_k}{Y_0} \right)^{-\frac{1}{\eta}}$ or $P_k = \left( \frac{Y_k}{Y_0} \right)^{-\frac{1}{\eta}}$ since $P_0 = 1$. Note also that $\frac{Y_k}{Y_0}$ presents the production level of good $k$ (say orange) in unit of $Y_0$ (say apple). Utilizing the same bilateral relations between the sector 0 and each sector from the -continuum, the production function (15) turns out

$$Y_j = Y_0 \left[ \int_{k \in J} X_{j,k} \left( P_k \right)^{1-\eta} \, dk \right]^\frac{\eta}{\eta-1}.$$

Observe that $Y_j = Y_0 \left[ \int_{k \in J} X_{j,k} \left( P_k \right)^{1-\eta} \, dk \right]^\frac{\eta}{\eta-1}$ and $P_j = \left( \frac{Y_k}{Y_0} \right)^{-\frac{1}{\eta}}$. 
gives rise to \( P_j = \left[ \int_{k \in J} X_{j,k}(P_k)^{1-\eta} dk \right]^{\frac{1}{1-\eta}} \) when \( \alpha = 1 \). Recall (4) and (6), which tell us that the aggregate price of input basket used by the rep \( j \) equals the market price \( P_j \), which is given by \( P_j = \left[ \int_{k \in J} X_{j,k}(P_k)^{1-\eta} dk \right]^{\frac{1}{1-\eta}} \) at the rep’s optimal input demand (5) subject to the production function (1). Having \( \alpha = 1 \) reflects the assumption of \textit{measure one} for each individual sector and for the size of \( J \). Therefore, (14) and (15) along \( \triangle_{j,k} = X_{j,k} \) implies there will be a market prices vector \( p \) that satisfies (1), (4), and (5) for every \( j \in J \). Consequently, for given \( X \) with \( x_{j,J} \neq 0 \) and \( x_{J,k} \neq 0 \) and for a given numeraire sector \( Y_0 > 0 \), there will be a general equilibrium having a prices vector such that \( z^d(p; X) = 0^J \) with numeraire price \( P_0 = 1 \), in which everyone \( j \in J \) but \( j \neq 0 \) is happy to produce a positive amount \( Y_j = Y_0 \left[ \int_{k \in J} X_{j,k}(P_k)^{1-\eta} dk \right]^{\frac{\eta}{\eta-1}} \) and sells and buys at \( P_j = \left[ \int_{k \in J} X_{j,k}(P_k)^{1-\eta} dk \right]^{\frac{1}{1-\eta}} \).

Finally, we recover market price and optimal production for the numeraire sector 0 as a function of \( (p; X) \):

\[
P_0 = \left[ \int_{k \in J} X_{0,k}(P_k)^{1-\eta} dk \right]^{\frac{1}{1-\eta}} \quad \text{and} \quad Y_0 = Y_0 \left[ \int_{k \in J} X_{0,k}(P_k)^{1-\eta} dk \right]^{\frac{\eta}{\eta-1}}
\]

Clearly, for \( Y_0 > 0 \), \( \int_{k \in J} X_{0,k}(P_k)^{1-\eta} dk = 1 \) and thus \( P_0 = 1 \). That is, \textit{any} sector with a positive production can serve as a numeraire, with normalization of its price at 1. We already know that, while the sector 0 serves as a numeraire, everyone is happy to produce and trade a positive amount. And thus, while someone else serves as a numeraire, the sector 0 is happy to produce and trade. Thus there is a general equilibrium in which everyone produces and trades at a given prices vector.
(2) **Proof of Zero Profits in a General Equilibrium:** We will show that every rep makes zero profits in a general equilibrium with a prices vector \( p \) such that \( z^d(p; X) = 0 \). By Lemma 1 and the first-order condition (4), we know that \( C_j = M_j = P_j \). Consequently, the rep \( j \) makes zero profits: \( \pi_j = P_j Y_j - C_j Y_j = P_j Y_j - P_j Y_j = 0 \). By definition, then, every market is in its partial equilibrium in a general equilibrium, and thus the first-order conditions that hold for the rep \( j \) will also hold for every firm in every market. Every rep makes zero profits.

**Proof of Lemma 2**

Let us rewrite \( M_j \) in (12) as

\[
M_j = \left[ \int_{k \in J} X_{j,k} (P_k + \gamma_{j,k})(P_k + \gamma_{j,k})^{-\eta} \right]^{\frac{1}{1-\eta}}.
\]

It is obvious that

\[
\int_{k \in J} X_{j,k} (P_k + \gamma_{j,k})(P_k + \gamma_{j,k})^{-\eta} \geq \int_{k \in J} X_{j,k} P_k (P_k + \gamma_{j,k})^{-\eta} \geq \int_{k \in J} X_{j,k} P_k (P_k + \gamma_{j,k})^{-\eta} \]

for any given \((p; X)\). In turn, since inputs are imperfect substitutes one another \((1 < \eta < \infty)\), we see

\[
\left[ \int_{k \in J} X_{j,k} (P_k + \gamma_{j,k})(P_k + \gamma_{j,k})^{-\eta} \right]^{\frac{1}{1-\eta}} = M_j \leq \bar{M}_j
\]

\[
= \left[ \int_{k \in J} X_{j,k} P_k (P_k + \gamma_{j,k})^{-\eta} \right]^{\frac{1}{1-\eta}}.
\]

We use this result that \( M_j \leq \bar{M}_j \) for any given \((p; X)\) and show that \( C_j \leq M_j \). Divide both sides of (13) by \( M_j \). We then have \( C_j / M_j = (M_j / \bar{M}_j)^{\eta-1} \). Since \( M_j \leq \bar{M}_j \) and \( 1 < \eta < \infty \), it is clear that \((M_j / \bar{M}_j)^{\eta-1} \leq 1 \) and thus \( C_j \leq M_j \).
References


