6. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\varepsilon = ir + (2i)R$ with $r = 0.200\ \Omega$ and $R = 2.00r$, we get

$$i_R = 2i = \frac{2\varepsilon}{r + 2R} = \frac{2(10.0\text{V})}{0.200\Omega + 2(0.400\Omega)} = 20.0\text{ A}.$$

(b) When connected in series $2\varepsilon - i_R r - i_R r - i_R R = 0$, or $i_R = 2\varepsilon/(2r + R)$. The result is

$$i_R = 2i = \frac{2\varepsilon}{2r + R} = \frac{2(10.0\text{V})}{2(0.200\Omega) + 0.400\Omega} = 25.0\text{ A}.$$

(c) They are in series arrangement, since $R > r$.

(d) If $R = r/2.00$, then for parallel connection,

$$i_R = 2i = \frac{2\varepsilon}{r + 2R} = \frac{2(10.0\text{V})}{0.200\Omega + 2(0.100\Omega)} = 50.0\text{ A}.$$

(e) For series connection, we have

$$i_R = 2i = \frac{2\varepsilon}{2r + R} = \frac{2(10.0\text{V})}{2(0.200\Omega) + 0.100\Omega} = 40.0\text{ A}.$$

(f) They are in parallel arrangement, since $R < r$.

19. First, we note in V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5 + R_6) = (2.80\ \text{A})(8.00\ \Omega + 4.00\ \Omega) = 33.6\ \text{V}.$$

The current through R_4 is then equal to $i_4 = V_4/R_4 = (33.6\ \text{V})/(16.0\ \Omega) = 2.10$ A. By the junction rule, the current in R_2 is

$$i_2 = i_4 + i_6 = 2.10\ \text{A} + 2.80\ \text{A} = 4.90\ \text{A},$$

so its voltage is

$$V_2 = (2.00\ \Omega)(4.90\ \text{A}) = 9.80\ \text{V}.$$

The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 9.80\ \text{V} + 33.6\ \text{V} = 43.4\ \text{V}$, implying that the current through it is $i_3 = V_3/(2.00\ \Omega) = 21.7$ A. The junction rule now gives the current in R_1 as

$$i_1 = i_2 + i_3 = 4.90\ \text{A} + 21.7\ \text{A} = 26.6\ \text{A},$$

implying that the voltage across it is $V_1 = (26.6\ \text{A})(2.00\ \Omega) = 53.2$ V. Therefore, by the loop rule,

$$\varepsilon = V_1 + V_3 = 53.2\ \text{V} + 43.4\ \text{V} = 96.6\ \text{V}.$$

29. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$\left(R_1 + R_2\right)i_1 = \left(R_x + R_s\right)\frac{R_1}{R_s}i_1 \quad \Rightarrow \quad R_x = \frac{R_2 R_s}{R_1}.$$

38. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R}=\frac{1}{R'}-\frac{1}{R_V} \quad \Rightarrow R'=\frac{RR_V}{R+R_V}.$$

The equivalent resistance of the circuit is $R_{\text{eq}} = R_A + R_0 + R' = R_A + R_0 + \dfrac{RR_V}{R+R_V}$.

(a) The ammeter reading is

$$i'=\frac{\varepsilon}{R_{\text{eq}}}=\frac{\varepsilon}{R_A+R_0+R_V\,R/(R+R_V)}=\frac{28.5\text{V}}{3.00\Omega+100\Omega+(300\Omega)\,(85.0\Omega)/(300\Omega+85.0\Omega)}$$
$$=0.168\text{ A}.$$

(b) The voltmeter reading is

$$V = \varepsilon - i'(R_A + R_0) = 28.5\text{ V} - (0.168\text{ A})(103.00\ \Omega) = 11.2\text{ V}.$$

(c) The apparent resistance is $R' = V/i' = (11.2\text{ V})/(0.168\text{ A}) = 66.2\ \Omega$.

(d) If R_V is increased, the difference between R and R' decreases. In fact, $R' \to R$ as $R_V \to \infty$.

42. The current in the circuit is

$$i = (150 \text{ V} - 50 \text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20 \text{ A}.$$

So from $V_Q + 150 \text{ V} - (2.0\ \Omega)i = V_P$, we get

$$V_Q = 100 \text{ V} + (2.0\ \Omega)(20 \text{ A}) - 150 \text{ V} = -10 \text{ V}.$$

47. **THINK** We have a multi-loop circuit with a capacitor that's being charged. Since at $t = 0$ the capacitor is completely uncharged, the current in the capacitor branch is as it would be if the capacitor were replaced by a wire.

EXPRESS Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$, the loop rule applied to the left-hand loop produces

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0.$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R.

ANALYZE (a) Solving the three simultaneous equations, we find

$$i_1 = \frac{2\varepsilon}{3R} = \frac{2(1.2\times10^3\text{ V})}{3(0.73\times10^6\,\Omega)} = 1.1\times10^{-3}\text{ A},$$

(b) $i_2 = \dfrac{\varepsilon}{3R} = \dfrac{1.2\times10^3\text{ V}}{3(0.73\times10^6\,\Omega)} = 5.5\times10^{-4}\text{ A},$

(c) and $i_3 = i_2 = 5.5\times10^{-4}\text{ A}.$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon - i_1 R_1 - i_1 R_2 = 0$.

(d) The solution is $i_1 = \dfrac{\varepsilon}{2R} = \dfrac{1.2\times10^3\text{ V}}{2(0.73\times10^6\,\Omega)} = 8.2\times10^{-4}\text{ A}$

(e) and $i_2 = i_1 = 8.2\times10^{-4}\text{ A}.$

(f) As stated before, the current in the capacitor branch is $i_3 = 0$.

We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1 R - i_2 R = 0$$

$$-\frac{q}{C} - i_3 R + i_2 R = 0.$$

We use the first equation to substitute for i_1 in the second and obtain

$$\varepsilon - 2i_2 R - i_3 R = 0.$$

Thus $i_2 = (\varepsilon - i_3 R)/2R$. We substitute this expression into the third equation above to obtain

$$-(q/C) - (i_3 R) + (\varepsilon/2) - (i_3 R/2) = 0.$$

Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{C\varepsilon}{2}\left(1 - e^{-2t/3RC}\right).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R} e^{-2t/3RC}.$$

The current in the center branch is

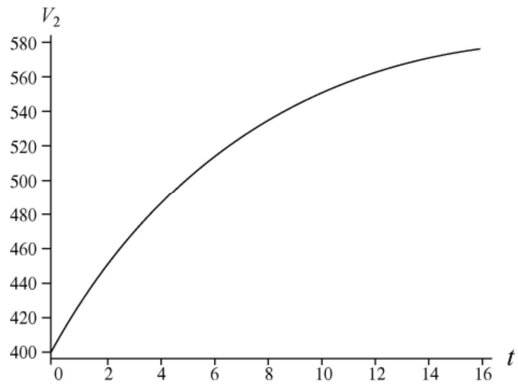
$$i_2(t) = \frac{\varepsilon}{2R} - \frac{i_3}{2} = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R} e^{-2t/3RC} = \frac{\varepsilon}{6R}\left(3 - e^{-2t/3RC}\right)$$

and the potential difference across R_2 is $V_2(t) = i_2 R = \frac{\varepsilon}{6}\left(3 - e^{-2t/3RC}\right)$.

(g) For $t = 0, e^{-2t/3RC} = 1$ and $V_2 = \varepsilon/3 = \left(1.2\times10^3\,\text{V}\right)/3 = 4.0\times10^2\,\text{V}$.

(h) For $t = \infty, e^{-2t/3RC} \to 0$ and $V_2 = \varepsilon/2 = \left(1.2\times20^3\,\text{V}\right)/2 = 6.0\times10^2\,\text{V}$.

(i) A plot of V_2 as a function of time is shown in the following graph.



LEARN A capacitor that is being charged initially behaves like an ordinary connecting wire relative to the charging current. However, a long time later after it's fully charged, it acts like a broken wire.