38. From the value of L in the graph when $\beta = 0$, we infer that L_0 in Eq. 37-13 is 0.80 m. Thus, that equation (which describes the curve in Fig. 37-23) with SI units understood becomes

$$L = L_0\sqrt{1-(v/c)^2} = (0.80\,\text{m})\sqrt{1-\beta^2}\ .$$

If we set $\beta = 0.90$ in this expression, we obtain approximately 0.35 m for L.

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question that has occasionally precipitated debates among professional physicists.

4. Due to the time-dilation effect, the time between initial and final ages for the daughter is longer than the four years experienced by her father:

$$t_{f\,\text{daughter}} - t_{i\,\text{daughter}} = \gamma(4.000\text{ y})$$

where γ is the Lorentz factor (Eq. 37-8). Letting T denote the age of the father, then the conditions of the problem require

$$T_i = t_{i\,\text{daughter}} + 20.00\text{ y}, \quad T_f = t_{f\,\text{daughter}} - 20.00\text{ y} .$$

Since $T_f - T_i = 4.000$ y, then these three equations combine to give a single condition from which γ can be determined (and consequently v):

$$44 = 4\gamma \quad \Rightarrow \quad \gamma = 11 \quad \Rightarrow \quad \beta = \frac{2\sqrt{30}}{11} = 0.9959.$$

5. In the laboratory, it travels a distance $d = 0.00105$ m $= vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 37-7 to relate t to the proper lifetime of the particle t_0:

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}} \quad \Rightarrow \quad t_0 = t\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1-0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s = 0.446 ps.

6. From the value of Δt in the graph when $\beta = 0$, we infer than Δt_0 in Eq. 37-9 is 8.0 s. Thus, that equation (which describes the curve in Fig. 37-22) becomes

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} = \frac{8.0\text{ s}}{\sqrt{1-\beta^2}} .$$

If we set $\beta = 0.98$ in this expression, we obtain approximately 40 s for Δt.

7. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

19. (a) We take the flashbulbs to be at rest in frame *S*, and let frame *S′* be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 37-21) must be used. Let t_s be the time and x_s be the coordinate of the small flash, as measured in frame *S*. Then, the time of the small flash, as measured in frame *S′*, is

$$t'_s = \gamma\left(t_s - \frac{\beta\, x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328\,.$$

Similarly, let t_b be the time and x_b be the coordinate of the big flash, as measured in frame *S*. Then, the time of the big flash, as measured in frame *S′*, is

$$t'_b = \gamma\left(t_b - \frac{\beta\, x_b}{c}\right).$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in *S*), we find

$$\Delta t' = \frac{\gamma\beta(x_s - x_b)}{c} = \frac{(1.0328)(0.250)(30\times10^3\text{ m})}{3.00\times10^8\text{ m/s}} = 2.58\times10^{-5}\text{ s}$$

where $\Delta t' = t'_b - t'_s$.

(b) Since $\Delta t'$ is negative, t'_b is greater than t'_s . The small flash occurs first in *S′*.

20. From Eq. 2 in Table 37-2, we have

$$\Delta t = v\,\gamma\,\Delta x'/c^2 + \gamma\,\Delta t'.$$

The coefficient of $\Delta x'$ is the slope (4.0 μs/400 m) of the graph, and the last term involving $\Delta t'$ is the "y-intercept" of the graph. From the first observation, we can solve for $\beta = v/c = 0.949$ and consequently $\gamma = 3.16$. Then, from the second observation, we find

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{2.00\times10^{-6}\text{s}}{3.16} = 6.3\times10^{-7}\text{s}\,.$$

21. (a) Using Eq. 2′ of Table 37-2, we have

29. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at 0.35*c* then an observer in Galaxy A should see our galaxy move away from him at 0.35*c*, or 0.35 in multiple of *c*.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 37-29, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$\frac{u'}{c} = \frac{u/c - v/c}{1 - uv/c^2} = \frac{(-0.35) - 0.35}{1 - (-0.35)(0.35)} = -0.62\,,$$

or $|u'/c| = 0.62$.

30. Using the notation of Eq. 37-29 and taking "away" (from us) as the positive direction, the problem indicates $v = +0.4c$ and $u = +0.8c$ (with 3 significant figures understood). We solve for the velocity of Q_2 relative to Q_1 (in multiple of *c*):

$$\frac{u'}{c} = \frac{u/c - v/c}{1 - uv/c^2} = \frac{0.8 - 0.4}{1 - (0.8)(0.4)} = 0.588$$

in a direction away from Earth.

31. Let S be the reference frame of the micrometeorite, and S' be the reference frame of the spaceship. We assume S to be moving in the $+x$ direction. Let u be the velocity of the micrometeorite as measured in S and v be the velocity of S' relative to S, the velocity of the micrometeorite as measured in S' can be solved by using Eq. 37-29:

$$u = \frac{u' + v}{1 + u'v/c^2} \;\Rightarrow\; u' = \frac{u - v}{1 - uv/c^2}.$$

The problem indicates that $v = -0.82c$ (spaceship velocity) and $u = +0.82c$ (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.82c - (-0.82c)}{1 - (0.82)(-0.82)} = 0.98c$$

or 2.94×10^8 m/s. Using Eq. 37-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t = \frac{d}{u'} = \frac{350\,\text{m}}{2.94 \times 10^8\ \text{m/s}} = 1.2 \times 10^{-6}\ \text{s}\,.$$

Note: The classical Galilean transformation would have given

$$u' = u - v = 0.82c - (-0.82c) = 1.64c,$$

which exceeds c and therefore, is physically impossible.

32. The figure shows that $u' = 0.80c$ when $v = 0$. We therefore infer (using the notation of Eq. 37-29) that $u = 0.80c$. Now, u is a fixed value and v is variable, so u' as a function of v is given by

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.80c - v}{1 - (0.80)\,v/c}$$

which is Eq. 37-29 rearranged so that u' is isolated on the left-hand side. We use this expression to answer parts (a) and (b).

(a) Substituting $v = 0.90c$ in the expression above leads to $u' = -0.357c \approx -0.36c$.

(b) Substituting $v = c$ in the expression above leads to $u' = -c$ (regardless of the value of u).

33. (a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c\,.$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781\ \text{ly}\,.$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625\text{c}} = 1.25\ \text{y}\,.$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c\,.$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.60\ \text{y}\,.$$

(c) Measured in system *S*, the length of the armada is

$$L=\frac{L_0}{\gamma}=1.0\,\text{ly}\sqrt{1-(0.80)^2}=0.60\ \text{ly}\ ,$$

so the length of the trip is

$$t=\frac{L}{v_m-v_a}=\frac{0.60\,\text{ly}}{0.95c-0.80c}=4.00\ \text{y}\ .$$

34. We use the transverse Doppler shift formula, Eq. 37-37: $f=f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda}=\frac{1}{\lambda_0}\sqrt{1-\beta^2}.$$

We solve for $\lambda-\lambda_0$:

$$\lambda-\lambda_0=\lambda_0\left(\frac{1}{\sqrt{1-\beta^2}}-1\right)=(589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}}-1\right]=+2.97\,\text{nm}\ .$$

35. The spaceship is moving away from Earth, so the frequency received is given directly by Eq. 37-31. Thus,

$$f=f_0\sqrt{\frac{1-\beta}{1+\beta}}=(100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}}=22.9\,\text{MHz}\ .$$

36. (a) Equation 37-36 leads to a speed of

$$v=\frac{\Delta\lambda}{\lambda}c=(0.004)(3.0\times10^8\,\text{m/s})=1.2\times10^6\ \text{m/s}\approx1\times10^6\ \text{m/s}.$$

(b) The galaxy is receding.

37. We obtain

$$v=\frac{\Delta\lambda}{\lambda}c=\left(\frac{620\ \text{nm}-540\ \text{nm}}{620\ \text{nm}}\right)c=0.13c.$$

38. (a) Equation 37-36 leads to

$$v=\frac{\Delta\lambda}{\lambda}c=\frac{12.00\,\text{nm}}{513.0\,\text{nm}}(2.998\times10^8\,\text{m/s})=7.000\times10^6\ \text{m/s}.$$

$$\gamma = \frac{E}{mc^2} = \frac{1.35\times10^5\ \text{MeV}}{139.6\ \text{MeV}} = 967.05.$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma\Delta t_0 = (967.1)(35.0\times10^{-9}\ \text{s}) = 3.385\times10^{-5}\ \text{s},$$

and the distance it travels is

$$d \approx c\Delta t = (2.998\times10^8\ \text{m/s})(3.385\times10^{-5}\ \text{s}) = 1.015\times10^4\ \text{m} = 10.15\ \text{km}$$

where we have approximated its speed as c (note: its speed can be found by solving Eq. 37-8, which gives $v = 0.9999995c$; this more precise value for v would not significantly alter our final result). Thus, the altitude at which the pion decays is 120 km – 10.15 km = 110 km.

46. (a) Squaring Eq. 37-47 gives

$$E^2 = (mc^2)^2 + 2mc^2K + K^2$$

which we set equal to Eq. 37-55. Thus,

$$(mc^2)^2 + 2mc^2K + K^2 = (pc)^2 + (mc^2)^2 \ \Rightarrow\ m = \frac{(pc)^2 - K^2}{2Kc^2}.$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \rightarrow \frac{(mvc)^2 - \left(\frac{1}{2}mv^2\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} \approx \frac{(mvc)^2}{2\left(\frac{1}{2}mv^2\right)c^2} = m.$$

(c) Here, pc = 121 MeV, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6\ \text{MeV}/\text{c}^2.$$

Now, the mass of the electron (see Table 37-3) is m_e = 0.511 MeV/c^2, so our result is roughly 207 times bigger than an electron mass, i.e., $m/m_e \approx 207$. The particle is a muon.

47. The energy equivalent of one tablet is

$$mc^2 = (320\times10^{-6}\ \text{kg})\ (3.00\times10^8\ \text{m/s})^2 = 2.88\times10^{13}\ \text{J}.$$