5. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^\infty EN(E)P(E)dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n}E_F^{5/2} \ .$$

Now

$$n = \int_0^\infty N(E)P(E)dE = C\int_0^{E_F} E^{1/2}dE = \frac{2C}{3}E_F^{3/2}.$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}} = \left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right) = \frac{3}{5}E_F \ .$$

13. **THINK** According to Appendix F the molar mass of silver is M = 107.870 g/mol and the density is ρ = 10.49 g/cm^3. Silver is monovalent.

EXPRESS The mass of a silver atom is, dividing the molar mass by Avogadro's number:

$$M_0 = \frac{M}{N_A} = \frac{107.870\times10^{-3}\text{ kg/mol}}{6.022\times10^{23}\text{ mol}^{-1}} = 1.791\times10^{-25}\text{ kg} .$$

Since silver is monovalent, there is one valence electron per atom (see Eq. 41-2).

ANALYZE (a) The number density is

$$n = \frac{\rho}{M_0} = \frac{10.49\times10^{-3}\text{ kg/m}^3}{1.791\times10^{-25}\text{ kg}} = 5.86\times10^{28}\text{ m}^{-3} .$$

This is the same as the number density of conduction electrons.

(b) The Fermi energy is

$$E_F = \frac{0.121h^2}{m}n^{2/3} = \frac{(0.121)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{9.109\times10^{-31}\text{ kg}} = (5.86\times10^{28}\text{ m}^{-3})^{2/3}$$
$$= 8.80\times10^{-19}\text{ J} = 5.49\text{eV}.$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80\times10^{-19}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 1.39\times10^6\text{ m/s} .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{mv_F} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.109\times10^{-31}\text{ kg})(1.39\times10^6\text{ m/s})} = 5.22\times10^{-10}\text{ m}.$$

LEARN Once the number density of conduction electrons is known, the Fermi energy for a particular metal can be calculated using Eq. 41-9.

21. (a) The probability that a state with energy E is occupied is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is E = 1.11 eV. Furthermore,

$$kT = (8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K}) = 0.02586 \text{ eV}.$$

For pure silicon, E_F = 0.555 eV and

$$(E - E_F)/kT = (0.555 \text{ eV})/(0.02586 \text{ eV}) = 21.46.$$

Thus,

$$P(E) = \frac{1}{e^{21.46} + 1} = 4.79 \times 10^{-10}.$$

(b) For the doped semiconductor,

$$(E - E_F)/kT = (0.11 \text{ eV})/(0.02586 \text{ eV}) = 4.254$$

and

$$P(E) = \frac{1}{e^{4.254} + 1} = 1.40 \times 10^{-2}.$$

(c) The energy of the donor state, relative to the top of the valence band, is 1.11 eV – 0.15 eV = 0.96 eV. The Fermi energy is 1.11 eV – 0.11 eV = 1.00 eV. Hence,

$$(E - E_F)/kT = (0.96 \text{ eV} - 1.00 \text{ eV})/(0.02586 \text{ eV}) = -1.547$$

and

$$P(E) = \frac{1}{e^{-1.547} + 1} = 0.824.$$

37. **THINK** The valence band and the conduction band are separated by an energy gap.

EXPRESS Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength.

ANALYZE (a) Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\,\text{m/s})}{(5.5\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = 2.26\times10^{-7}\ \text{m} = 226\,\text{nm}\ .$$

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

LEARN Note that photons from other transitions have a greater energy, so their waves have shorter wavelengths.