9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = M\,gH$. Its final kinetic energy (as it leaves the track) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 11-5) and its final potential energy is $M\,gh$. Here we use v to denote the speed of its center of mass and ω is its angular speed — at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \frac{2}{5}MR^2$ (Table 10-2(f)), conservation of energy leads to

$$MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh$$
$$= \frac{7}{10}Mv^2 + Mgh.$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7}g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\ \text{m/s}^2\right)\left(6.0\ \text{m} - 2.0\ \text{m}\right)} = 7.48\ \text{m/s}.$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt \ \text{and}\ y = -\frac{1}{2}gt^2.$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48\ \text{m/s})\sqrt{\frac{2(2.0\ \text{m})}{9.8\ \text{m/s}^2}} = 4.8\ \text{m}.$$

16. Using energy conservation with Eq. 11-5 and solving for the rotational inertia (about the center of mass), we find

$$I_{com} = 2MhR^2/r - MR^2 = MR^2[2g(H - h)/v^2 - 1]\ .$$

Thus, using the β notation suggested in the problem, we find

$$\beta = 2g(H - h)/v^2 - 1.$$

To proceed further, we need to find the center of mass speed v, which we do using the projectile motion equations of Chapter 4. With $v_{oy} = 0$, Eq. 4-22 gives the time-of-flight as $t = \sqrt{2h/g}$. Then Eq. 4-21 (squared, and using d for the horizontal displacement) gives $v^2 = gd^2/2h$. Plugging this into our expression for β gives

$$2g(H - h)/v^2 - 1 = 4h(H - h)/d^2 - 1.$$

Therefore, with the values given in the problem, we find $\beta = 0.25$.

26. We note that the component of v perpendicular to r has magnitude $v \sin \theta_2$ where $\theta_2 = 30°$. A similar observation applies to $\vec{F}$.

(a) Eq. 11-20 leads to

$$\ell = rmv_{\perp} = (3.0 \text{ m})(2.0 \text{ kg})(4.0 \text{ m/s})\sin 30° = 12 \text{ kg}\cdot\text{m}^2/\text{s}.$$

(b) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, or along the +z axis, perpendicular to the plane of the figure.

(c) Similarly, Eq. 10-38 leads to

$$\tau = rF \sin \theta_2 = (3.0 \text{ m})(2.0 \text{ N})\sin 30° = 3.0 \text{N}\cdot\text{m}.$$

(d) Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page, or along the +z axis, perpendicular to the plane of the figure.

44. So that we don't get confused about ± signs, we write the angular *speed* to the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I\left|\omega_0\right| = -\left(mR^2 + I\right)\left|\omega_f\right|.$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I\,|\omega_0|}{mR^2 + I} = \frac{(0.17\text{ kg})(2.0\text{ m/s})(0.15\text{ m}) - (5.0\times10^{-3}\text{ kg}\cdot\text{m}^2)(2.8\text{ rad/s})}{(5.0\times10^{-3}\text{ kg}\cdot\text{m}^2) + (0.17\text{ kg})(0.15\text{ m})^2}$$
$$= 4.2\text{ rad/s}.$$

(b) No, $K_f \neq K_i$ and — if desired — we can solve for the difference:

$$K_i - K_f = \frac{mI}{2}\,\frac{v^2 + \omega_0^2 R^2 + 2Rv\left|\omega_0\right|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" — that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

54. We denote the cat with subscript 1 and the ring with subscript 2. The cat has a mass $m_1 = M/4$, while the mass of the ring is $m_2 = M = 8.00$ kg. The moment of inertia of the ring is $I_2 = m_2(R_1^2 + R_2^2)/2$ (Table 10-2), and $I_1 = m_1 r^2$ for the cat, where r is the perpendicular distance from the axis of rotation.

Initially the angular momentum of the system consisting of the cat (at $r = R_2$) and the ring is

$$L_i = m_1 v_{1i} r_{1i} + I_2 \omega_{2i} = m_1 \omega_0 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_0 = m_1 R_2^2 \omega_0 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right].$$

After the cat has crawled to the inner edge at $r = R_1$ the final angular momentum of the system is

$$L_f = m_1 \omega_f R_1^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\omega_f = m_1 R_1^2 \omega_f \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)\right].$$

Then from $L_f = L_i$ we obtain

$$\frac{\omega_f}{\omega_0} = \left(\frac{R_2}{R_1}\right)^2 \frac{1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)}{1 + \frac{1}{2}\frac{m_2}{m_1}\left(1 + \frac{R_2^2}{R_1^2}\right)} = (2.0)^2 \frac{1 + 2(0.25 + 1)}{1 + 2(1 + 4)} = 1.273.$$

Thus, $\omega_f = 1.273\omega_0$. Using $\omega_0 = 8.00$ rad/s, we have $\omega_f = 10.2$ rad/s. By substituting $I = L/\omega$ into $K = I\omega^2/2$, we obtain $K = L\omega/2$. Since $L_i = L_f$, the kinetic energy ratio becomes

$$\frac{K_f}{K_i} = \frac{L_f \omega_f / 2}{L_i \omega_i / 2} = \frac{\omega_f}{\omega_0} = 1.273.$$

which implies $\Delta K = K_f - K_i = 0.273 K_i$. The cat does positive work while walking toward the center of the ring, increasing the total kinetic energy of the system.

Since the initial kinetic energy is given by

$$K_i = \frac{1}{2}\left[m_1 R_2^2 + \frac{1}{2} m_2 (R_1^2 + R_2^2)\right]\omega_0^2 = \frac{1}{2} m_1 R_2^2 \omega_0^2 \left[1 + \frac{1}{2}\frac{m_2}{m_1}\left(\frac{R_1^2}{R_2^2} + 1\right)\right]$$

$$= \frac{1}{2}(2.00 \text{ kg})(0.800 \text{ m})^2 (8.00 \text{ rad/s})^2 [1 + (1/2)(4)(0.5^2 + 1)]$$

$$= 143.36 \text{ J},$$

the increase in kinetic energy is

$$\Delta K = (0.273)(143.36 \text{ J}) = 39.1 \text{ J}.$$

60. (a) With $r = 0.75$ m, we obtain $I = 0.060 + (0.601)r^2 = 0.40\ \text{kg}\cdot\text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\ell_0 = L_f \quad\Rightarrow\quad mv_0 r = I\omega \quad\Rightarrow\quad (0.001)v_0(0.75) = (0.40)(4.5)$$

which leads to $v_0 = 2.4 \times 10^3$ m/s.