

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are  $n_1$  loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1v_1/2f,$$

where  $\lambda_1$  is the wavelength and  $v_1$  is the wave speed in that section. In this consideration, we have substituted  $\lambda_1 = v_1/f$ , where  $f$  is the frequency. Thus  $f = n_1v_1/2L_1$ . A similar expression holds for the steel section:  $f = n_2v_2/2L_2$ . Since the frequency is the same for the two sections,  $n_1v_1/L_1 = n_2v_2/L_2$ . Now the wave speed in the aluminum section is given by  $v_1 = \sqrt{\tau/\mu_1}$ , where  $\mu_1$  is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by  $m_1 = \rho_1AL_1$ , where  $\rho_1$  is the mass density (mass per unit volume) for aluminum and  $A$  is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1AL_1/L_1 = \rho_1A$$

and  $v_1 = \sqrt{\tau/\rho_1A}$ . A similar expression holds for the wave speed in the steel section:  $v_2 = \sqrt{\tau/\rho_2A}$ . We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to  $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$ , where  $A$  has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\text{ m})\sqrt{7.80\times 10^3\text{ kg/m}^3}}{(0.600\text{ m})\sqrt{2.60\times 10^3\text{ kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are  $n_1 = 2$  and  $n_2 = 5$ . The frequency is

$$f = n_1v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1A}.$$

The tension is provided by the hanging block and is  $\tau = mg$ , where  $m$  is the mass of the block. Thus,

$$f = \frac{n_1}{2L_1} \sqrt{\frac{mg}{\rho_1A}} = \frac{2}{2(0.600\text{ m})} \sqrt{\frac{(10.0\text{ kg})(9.80\text{ m/s}^2)}{(2.60\times 10^3\text{ kg/m}^3)(1.00\times 10^{-6}\text{ m}^2)}} = 324\text{ Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

37. (a) Using the phasor technique, we think of these as two “vectors” (the first of “length” 4.6 mm and the second of “length” 5.60 mm) separated by an angle of  $\phi = 0.8\pi$  radians (or  $144^\circ$ ). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to  $88.8^\circ$  (or 1.55 rad).

(c) Clearly, it should be “in phase” with the result we just calculated, so its phase angle relative to the first phasor should be also  $88.8^\circ$  (or 1.55 rad).

10. (a) The amplitude is  $y_m = 6.0$  cm.

(b) We find  $\lambda$  from  $2\pi/\lambda = 0.020\pi$ .  $\lambda = 1.0 \times 10^2$  cm.

(c) Solving  $2\pi f = \omega = 4.0\pi$ , we obtain  $f = 2.0$  Hz.

(d) The wave speed is  $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0 \times 10^2$  cm/s.

(e) The wave propagates in the  $-x$  direction, since the argument of the trig function is  $kx + \omega t$  instead of  $kx - \omega t$  (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of  $y$ ) is

$$u_{\max} = 2\pi f y_m = (4.0\pi \text{ s}^{-1})(6.0 \text{ cm}) = 75 \text{ cm/s}.$$

(g)  $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0 \text{ cm}.$

25. (a) The wave speed at any point on the rope is given by  $v = \sqrt{\tau/\mu}$ , where  $\tau$  is the tension at that point and  $\mu$  is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance  $y$  from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by  $\mu gy$ , so the tension is  $\tau = \mu gy$ . The wave speed is  $v = \sqrt{\mu gy / \mu} = \sqrt{gy}$ .

(b) The time  $dt$  for the wave to move past a length  $dy$ , a distance  $y$  from the bottom end, is  $dt = dy/v = dy/\sqrt{gy}$  and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}} \Big|_0^L = 2\sqrt{\frac{L}{g}}.$$

30. The wave  $y(x,t) = (4.00 \text{ mm}) h[(30 \text{ m}^{-1})x + (6.0 \text{ s}^{-1})t]$  is of the form  $h(kx - \omega t)$  with angular wave number  $k = 30 \text{ m}^{-1}$  and angular frequency  $\omega = 6.0 \text{ rad/s}$ . Thus, the speed of the wave is

$$v = \omega / k = (6.0 \text{ rad/s}) / (30 \text{ m}^{-1}) = 0.20 \text{ m/s}.$$

33. (a) The amplitude of the second wave is  $y_m = 9.00$  mm, as stated in the problem.

(b) The figure indicates that  $\lambda = 40$  cm = 0.40 m, which implies that the angular wave number is  $k = 2\pi/0.40 = 16$  rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is  $v = dx/t = (56.0$  cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = k v = 1100 \text{ rad/s} = 1.1 \times 10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the  $-x$  direction) of equal amplitude  $y_m$ . The amplitude of their resultant wave, as shown in the figure, is  $y'_m = 4.00$  mm. Equation 16-52 applies:

$$y'_m = 2y_m \cos\left(\frac{1}{2}\phi_2\right) \Rightarrow \phi_2 = 2 \cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in  $y = y_m \sin(kx \pm \omega t + \phi)$ , we recall the discussion in section 16-5, where it was shown that sinusoidal waves traveling in the  $-x$  direction are of the form  $y = y_m \sin(kx + \omega t + \phi)$ . Here,  $\phi$  should be thought of as the phase *difference* between the two waves (that is,  $\phi_1 = 0$  for wave 1 and  $\phi_2 = 2.69$  rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100t) \quad \text{and} \quad y_2 = (0.00900)\sin(16x + 1100t + 2.7).$$