11. We write

$\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters.

(a) The x and the y components of $\vec{r}$ are $r_x = a_x + b_x$ = (4.0 m) – (13 m) = –9.0 m and $r_y = a_y + b_y$ = (3.0 m) + (7.0 m) = 10 m, respectively. Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\text{i}} + (10\,\text{m})\,\hat{\text{j}}$.

(b) The magnitude of $\vec{r}$ is

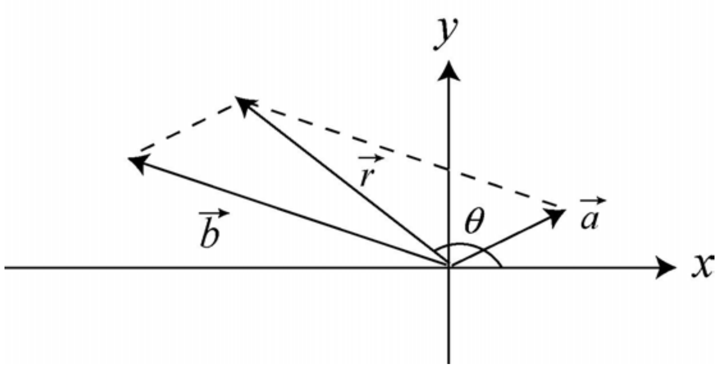
$$r = |\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0\text{ m})^2 + (10\text{ m})^2} = 13\text{ m}\,.$$

(c) The angle between the resultant and the $+x$ axis is given by

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{10.0\text{ m}}{-9.0\text{ m}}\right) = -48^\circ \text{ or } 132^\circ.$$

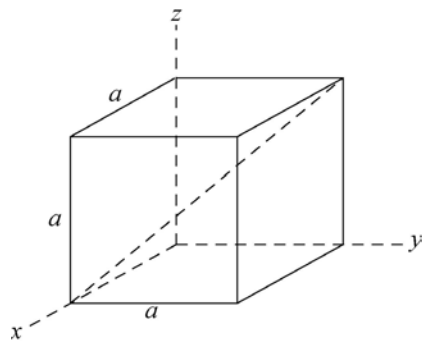
Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

The addition of the two vectors is depicted in the figure below (not to scale). Indeed, we expect $\vec{r}$ to be in the second quadrant.



31. (a) As can be seen from Figure 3-30, the point diametrically opposite the origin (0,0,0) has position vector $a\,\hat{\text{i}} + a\,\hat{\text{j}} + a\,\hat{\text{k}}$ and this is the vector along the "body diagonal."

(b) From the point (a, 0, 0), which corresponds to the position vector $a\,\hat{\text{i}}$, the diametrically opposite point is (0, a, a) with the position vector $a\,\hat{\text{j}} + a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $-a\hat{\text{i}} + a\hat{\text{j}} + a\hat{\text{k}}$.



(c) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\text{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\,\hat{\text{i}} + a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $a\,\hat{\text{i}} - a\,\hat{\text{j}} + a\,\hat{\text{k}}$.

(d) If the starting point is (a, a, 0) with the corresponding position vector $a\,\hat{\text{i}} + a\,\hat{\text{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\,\hat{\text{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\text{i}} - a\,\hat{\text{j}} + a\,\hat{\text{k}}$.

(e) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\text{i}} + a\,\hat{\text{j}} + a\,\hat{\text{k}}$. We may think of it as the sum of the vector $a\,\hat{\text{i}}$ parallel to the x axis and the vector $a\,\hat{\text{j}} + a\,\hat{\text{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(f) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

37. We apply Eq. 3-30 and Eq.3-23. If a vector-capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8.0\,\hat{\mathrm{i}} + 5.0\,\hat{\mathrm{j}} + 6.0\,\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = (3.0)\,(-8.0) + (3.0)(5.0) + (-2.0)\,(6.0) = -21.$$

(b) We note that $\vec{b} + \vec{c} = 1.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$. Thus,

$$\vec{a}\cdot(\vec{b}+\vec{c}) = (3.0)\,(1.0) + (3.0)\,(-2.0) + (-2.0)\,(3.0) = -9.0.$$

(c) Finally,

$$\begin{aligned}\vec{a}\times(\vec{b}+\vec{c}) &= [(3.0)(3.0)-(-2.0)(-2.0)]\,\hat{\mathrm{i}} + [(-2.0)(1.0)-(3.0)(3.0)]\,\hat{\mathrm{j}} \\ &\quad + [(3.0)(-2.0)-(3.0)(1.0)]\,\hat{\mathrm{k}} \\ &= 5\hat{\mathrm{i}} - 11\hat{\mathrm{j}} - 9\hat{\mathrm{k}}\end{aligned}$$

38. Using the fact that

$$\hat{\mathrm{i}} \times \hat{\mathrm{j}} = \hat{\mathrm{k}}, \quad \hat{\mathrm{j}} \times \hat{\mathrm{k}} = \hat{\mathrm{i}}, \quad \hat{\mathrm{k}} \times \hat{\mathrm{i}} = \hat{\mathrm{j}}$$

we obtain

$$2\vec{A} \times \vec{B} = 2\left(2.00\hat{\mathrm{i}} + 3.00\hat{\mathrm{j}} - 4.00\hat{\mathrm{k}}\right) \times \left(-3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}} + 2.00\hat{\mathrm{k}}\right) = 44.0\hat{\mathrm{i}} + 16.0\hat{\mathrm{j}} + 34.0\hat{\mathrm{k}}.$$

Next, making use of

$$\begin{aligned} \hat{\mathrm{i}} \cdot \hat{\mathrm{i}} &= \hat{\mathrm{j}} \cdot \hat{\mathrm{j}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{k}} = 1 \\ \hat{\mathrm{i}} \cdot \hat{\mathrm{j}} &= \hat{\mathrm{j}} \cdot \hat{\mathrm{k}} = \hat{\mathrm{k}} \cdot \hat{\mathrm{i}} = 0 \end{aligned}$$

we have

$$\begin{aligned} 3\vec{C} \cdot \left(2\vec{A} \times \vec{B}\right) &= 3\left(7.00\hat{\mathrm{i}} - 8.00\hat{\mathrm{j}}\right) \cdot \left(44.0\hat{\mathrm{i}} + 16.0\hat{\mathrm{j}} + 34.0\hat{\mathrm{k}}\right) \\ &= 3[(7.00)(44.0) + (-8.00)(16.0) + (0)(34.0)] = 540. \end{aligned}$$

52. The three vectors are

$$\vec{d}_1 = 4.0\hat{\mathrm{i}} + 5.0\hat{\mathrm{j}} - 6.0\hat{\mathrm{k}}$$
$$\vec{d}_2 = -1.0\hat{\mathrm{i}} + 2.0\hat{\mathrm{j}} + 3.0\hat{\mathrm{k}}$$
$$\vec{d}_3 = 4.0\hat{\mathrm{i}} + 3.0\hat{\mathrm{j}} + 2.0\hat{\mathrm{k}}$$

(a) $\vec{r} = \vec{d}_1 - \vec{d}_2 + \vec{d}_3 = (9.0\ \mathrm{m})\hat{\mathrm{i}} + (6.0\ \mathrm{m})\hat{\mathrm{j}} + (-7.0\ \mathrm{m})\hat{\mathrm{k}}$.

(b) The magnitude of $\vec{r}$ is $|\vec{r}| = \sqrt{(9.0\ \mathrm{m})^2 + (6.0\ \mathrm{m})^2 + (-7.0\ \mathrm{m})^2} = 12.9\ \mathrm{m}$. The angle between $\vec{r}$ and the *z*-axis is given by

$$\cos\theta = \frac{\vec{r}\cdot\hat{\mathrm{k}}}{|\vec{r}\,|} = \frac{-7.0\ \mathrm{m}}{12.9\ \mathrm{m}} = -0.543$$

which implies $\theta = 123°$.

(c) The component of $\vec{d}_1$ along the direction of $\vec{d}_2$ is given by $d_{\parallel} = \vec{d}_1\cdot\hat{\mathrm{u}} = d_1\cos\varphi$ where φ is the angle between $\vec{d}_1$ and $\vec{d}_2$, and $\hat{\mathrm{u}}$ is the unit vector in the direction of $\vec{d}_2$. Using the properties of the scalar (dot) product, we have

$$d_{\parallel} = d_1\left(\frac{\vec{d}_1\cdot\vec{d}_2}{d_1 d_2}\right) = \frac{\vec{d}_1\cdot\vec{d}_2}{d_2} = \frac{(4.0)(-1.0)+(5.0)(2.0)+(-6.0)(3.0)}{\sqrt{(-1.0)^2+(2.0)^2+(3.0)^2}} = \frac{-12}{\sqrt{14}} = -3.2\ \mathrm{m}.$$

(d) Now we are looking for $d_{\perp}$ such that $d_1^2 = (4.0)^2 + (5.0)^2 + (-6.0)^2 = 77 = d_{\parallel}^2 + d_{\perp}^2$. From (c), we have

$$d_{\perp} = \sqrt{77\ \mathrm{m}^2 - (-3.2\ \mathrm{m})^2} = 8.2\ \mathrm{m}.$$

This gives the magnitude of the perpendicular component (and is consistent with what one would get using Eq. 3-27), but if more information (such as the direction, or a full specification in terms of unit vectors) is sought then more computation is needed.

43. From the figure, we note that

$\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is $\theta + 90°$. In unit-vector notation, the three vectors can be written as

$$\vec{a} = a_x \hat{\mathrm{i}}$$
$$\vec{b} = b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}} = (b\cos\theta)\hat{\mathrm{i}} + (b\sin\theta)\hat{\mathrm{j}}$$
$$\vec{c} = c_x \hat{\mathrm{i}} + c_y \hat{\mathrm{j}} = [c\cos(\theta + 90°)]\hat{\mathrm{i}} + [c\sin(\theta + 90°)]\hat{\mathrm{j}}$$

The above expressions allow us to evaluate the components of the vectors.

(a) The x-component of $\vec{a}$ is $a_x = a \cos 0° = a = 3.00$ m.

(b) Similarly, the y-componnet of $\vec{a}$ is $a_y = a \sin 0° = 0$.

(c) The x-component of $\vec{b}$ is $b_x = b \cos 30° = (4.00 \text{ m}) \cos 30° = 3.46$ m,

(d) and the y-component is $b_y = b \sin 30° = (4.00 \text{ m}) \sin 30° = 2.00$ m.

(e) The x-component of $\vec{c}$ is $c_x = c \cos 120° = (10.0 \text{ m}) \cos 120° = –5.00$ m,

(f) and the y-component is $c_y = c \sin 30° = (10.0 \text{ m}) \sin 120° = 8.66$ m.

(g) The fact that $\vec{c} = p\vec{a} + q\vec{b}$ implies

$$\vec{c} = c_x \hat{\mathrm{i}} + c_y \hat{\mathrm{j}} = p(a_x \hat{\mathrm{i}}) + q(b_x \hat{\mathrm{i}} + b_y \hat{\mathrm{j}}) = (pa_x + qb_x)\hat{\mathrm{i}} + qb_y \hat{\mathrm{j}}$$

or

$$c_x = pa_x + qb_x, \qquad c_y = qb_y$$

Substituting the values found above, we have

$$-5.00 \text{ m} = p\,(3.00 \text{ m}) + q\,(3.46 \text{ m})$$
$$8.66 \text{ m} = q\,(2.00 \text{ m}).$$

Solving these equations, we find $p = –6.67$.

(h) Similarly, $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.