

6. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for dx/dt .

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt} (3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}) = (3.00 \text{ m/s})\hat{i} - (8.00t \text{ m/s})\hat{j}$$

(b) Evaluating this result at $t = 2.00$ s produces $\vec{v} = (3.00\hat{i} - 16.0\hat{j})$ m/s.

(c) The speed at $t = 2.00$ s is $v = |\vec{v}| = \sqrt{(3.00)^2 + (-16.0)^2} = 16.3$ m/s.

(d) And the angle of \vec{v} at that moment is one of the possibilities

$$\tan^{-1} \left(\frac{-16.0}{3.00} \right) = -79.4^\circ \text{ or } 101^\circ$$

where we choose the first possibility (79.4° measured clockwise from the $+x$ direction, or 281° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

38. In this projectile motion problem, we have $v_0 = v_x = \text{constant}$, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at $t = 2.5$ s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x = 19$ m/s.

(a) During $t = 5$ s, the horizontal motion is $x - x_0 = v_x t = 95$ m.

(b) Since $\sqrt{(19 \text{ m/s})^2 + v_{0y}^2} = 31$ m/s (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with $t = 2.5$ s, we can use $y_{\text{max}} - y_0 = v_{0y} t - \frac{1}{2} g t^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\text{max}} - y_0)$, or $y_{\text{max}} - y_0 = \frac{1}{2}(v_y + v_{0y})t$ to solve. Here we will use the latter:

$$y_{\text{max}} - y_0 = \frac{1}{2}(v_y + v_{0y})t \Rightarrow y_{\text{max}} = \frac{1}{2}(0 + 24.5 \text{ m/s})(2.5 \text{ s}) = 31 \text{ m}$$

where we have taken $y_0 = 0$ as the ground level.

57. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (toward the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83 \text{ m/s}^2$ east, then the center of the circle is *east* of this location. The distance is $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32 \text{ m}$.

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

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"(c) If the direction of \vec{a} experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

76. The destination is $\vec{D} = 800 \text{ km } \hat{j}$ where we orient axes so that +y points north and +x points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is $\vec{v}_{pg} = (400 \text{ km/h}) \hat{j}$. This must be the vector sum of the plane's velocity with respect to the air which has (x,y) components $(500\cos 70^\circ, 500\sin 70^\circ)$, and the velocity of the air (*wind*) relative to the ground \vec{v}_{ag} . Thus,

$$(400 \text{ km/h}) \hat{j} = (500 \text{ km/h}) \cos 70^\circ \hat{i} + (500 \text{ km/h}) \sin 70^\circ \hat{j} + \vec{v}_{ag}$$

which yields

$$\vec{v}_{ag} = (-171 \text{ km/h}) \hat{i} - (70.0 \text{ km/h}) \hat{j}.$$

(a) The magnitude of \vec{v}_{ag} is $|\vec{v}_{ag}| = \sqrt{(-171 \text{ km/h})^2 + (-70.0 \text{ km/h})^2} = 185 \text{ km/h}$.

(b) The direction of \vec{v}_{ag} is

$$\theta = \tan^{-1} \left(\frac{-70.0 \text{ km/h}}{-171 \text{ km/h}} \right) = 22.3^\circ \text{ (south of west).}$$

78. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep P relative to A at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) = (20.0 \text{ m/s})\hat{i} + (34.6 \text{ m/s})\hat{j}.$$

Similarly, the velocity of Jeep B relative to A at the instant is

$$\vec{v}_{BA} = (20.0 \text{ m/s})(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) = (17.3 \text{ m/s})\hat{i} + (10.0 \text{ m/s})\hat{j}.$$

Thus, the velocity of P relative to B is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{i} + 34.6\hat{j}) \text{ m/s} - (17.3\hat{i} + 10.0\hat{j}) \text{ m/s} = (2.68 \text{ m/s})\hat{i} + (24.6 \text{ m/s})\hat{j}.$$

(a) The magnitude of \vec{v}_{PB} is $|\vec{v}_{PB}| = \sqrt{(2.68 \text{ m/s})^2 + (24.6 \text{ m/s})^2} = 24.8 \text{ m/s}$.

(b) The direction of \vec{v}_{PB} is $\theta = \tan^{-1}[(24.6 \text{ m/s})/(2.68 \text{ m/s})] = 83.8^\circ$ north of east (or 6.2° east of north).

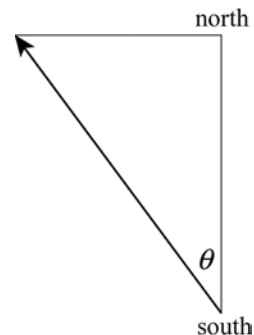
(c) The acceleration of P is

$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0^\circ \hat{i} + \sin 60.0^\circ \hat{j}) = (0.200 \text{ m/s}^2)\hat{i} + (0.346 \text{ m/s}^2)\hat{j},$$

and $\vec{a}_{PA} = \vec{a}_{PB}$. Thus, we have $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$.

(d) The direction is 60.0° north of east (or 30.0° east of north).

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82 \text{ m}) + (1.1 \text{ m/s})t$, where the t -dependent contribution is the distance that the river will carry the boat downstream during time t .



The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean “sum” of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0.$$

(b) We solve for t first and find a positive value: $t = 62.6 \text{ s}$.

(a) The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured “west of north”) is then given by

$$\theta = \tan^{-1} \left(\frac{82 + 1.1t}{200} \right) = \tan^{-1} \left(\frac{151}{200} \right) = 37^\circ.$$